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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

YEREVAN PHYSICS INSTITUTE

A.A.CHILINGARYAN,

DIMENSIONALITY ANALYSIS OF MULTIPARTICLE PRODUCTION AT HIGH ENERGIES

ЦНИИатоминформ

ЕРЕВАН - 1989

Նախնատիպ ԵՖԻ-1213(90)-89

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ԲԱԶՄԱԾԻՆ ԳՈՐԾԸՆԹԱՑՆԵՐԻ ՉԱՓՈՂԱԿԱՆ ՎԵՐԼՈՒԾՈՒԹՅՈՒՆ

Առաջարկվում է ալգորիթմ՝ բազմամասնիկ վերջնական վիճակների վերլուծության համար: Փորձառական տվյալներով հաշված Ռենյիի չափողականություններով, ինչպես այն հադրոնների բաշխումը ըստ սրընթացության ինտերվալների, ինչ մասնիկների բաշխումը T -չափանի իմպուլսային տարածությունում, կարելի է դատողություն անել մասնիկների կոռելյացվածության աստիճանի մասին, առանձնացնել պրոյեկցիաները և իմպուլսային տարածության այն տիրույթները, որտեղ նկատվում են հավանականական չափի սինգուլյարություն: Այս մեթոդը ստուգվել է հաշվողական փորձերի շարքում կոտորակային օբյեկտների Կանտորի քաղվածքներով, որոնք ստացվել են տարբեր ինվացյալ- և գրեթե- պատահական թվերի տվիչների օգնությամբ:

Երևանի Ֆիզիկայի ինստիտուտ

Երևան 1989

1. Phenomenological Description of Multiple Production

Significant increase of information about multiparticle final states produced in particle collisions with higher and higher energies make it urgent to develop non traditional methods of analysis of experimental data. By means of the parameters of the detected particles one can construct many joint and conditional probability distributions which are much more informative than the averaged characteristics [1].

Since there is still no general theory of strong interactions, a phenomenological approach to the ultra high energy collisions is widely used. One of the first theoretical generalizations of the multiple production processes is the KNO scaling, which predicts that at sufficiently high energies the distribution over hadron multiplicity P_n obeys the scaling

$$P_n \langle n \rangle = \Psi(z), \qquad z = n/\langle n \rangle , \tag{1.1}$$

where P_n is the probability to observe n hadrons in the final state, $\langle n \rangle$ is the mean multiplicity at a given energy.

Since the Poisson distribution

$$P_n = \langle n \rangle^n \, e^{-\langle n \rangle} / \, n! \tag{1.2}$$

describes the hadron multiplicity badly, there was proposed to use the negative binomial distribution and the Bose-Ein distribution, which supposes the presence of κ independent random sources with the same intensity:

$$P_n^{\langle k\rangle}\langle n\rangle = \Psi_k(z) = k^k z^{k-1} e^{-kz}/(k-1)! \qquad (1.3)$$

Carruthers has shown [2] that $\Psi_2(Z)$ describes the ISR and SPS data well.

Though the description of the nature of random sources meets difficulties (they are associated with the quark-gluon plasma density oscillations), recently, using the Bose-Einstein correlations, they succeeded in estimating the size of hadron sources [3]. The source size in $p\bar{p}$ collisions did not change when the energy changed from 0.9 to 2.2TeV in the c.m.s. (as was to be expected, if the KNO scaling was satisfied) and was in a linear dependence with the charged density in the pseudo-rapidity bin ($\Delta n/\Delta\eta$):

$$R_{Fermi} = 0.59 \pm 0.05 \; (\Delta n/\Delta\eta) \; . \qquad (1.4)$$

Recently the particles distribution in the rapidity windows became the object of great attention. Large fluctuations in some rapidity bins, which were found in experiments at colliders and in cosmic-ray physics [4], could not find any description in the frame of earlier suggested phenomenological mechanisms. Conclusions were drawn that the large fluctuations in the rapidity distributions reflect non-trivial fluctuations of the hadronic matter during collisions.

The instrument of investigation of non-trivial rapidity correlations till now is the study of the dependence of normalized moments of the rapidity distributions on the size of the rapidity bin [5]. Several modifications of the moments method are suggested:

$$C_q = \langle n^q \rangle / \langle n \rangle^q \quad , \qquad q = 1,2...,$$

$$C'_q = \langle (n - \langle n \rangle)^q \rangle \ / \ \langle n \rangle^q , \tag{1.5}$$

$$C''_q = \langle n(n-1)...(n-q+1)\rangle \ / \ \langle n \rangle^q \ ,$$

where q is the order of the normalized moments and $\langle \ \rangle$ means averaging over the rapidity bins.

Let us write down a more detailed expression of a normalized moment:

$$C_q(M) = 1/M \sum_{m=1}^{M} n_m^q \ / \langle n_M \rangle^q \ , \tag{1.6}$$

where M is the number of equal rapidity bins - $\delta_y = \Delta/M$; Δ usually is the interval (-2 - 2), i.e. $\delta_y = 4/M$; n_m is the number of hadrons falling into the m-th bin, $\langle n_m \rangle$ is the average bin population of events with multiplicity N, M is the number of bins.

Let us consider, following [6], how the normalized moments behave under assumption of absence of correlation and under very strong correlation. Consider the uniform bins distribution: $n_m = N/M$, m=i,...M. It is easily seen that for all q, $F_q(M)=1$. And if all the hadrons have fallen into the same bin, $n_m = N$ for some m=r and $n_m = 0$ for the remaining m, then

$$C_q(M) = M^{q-1} \quad , \tag{1.7}$$

i.e. at an extremal fluctuation the moments significantly increase with the number of bins. That is why the moments method sometimes is called a magnifier for exposure of non-uniformities. Rewrite (1.7) in somewhat different form and take its logarithm:

$$C_q(M) = (\Delta/\delta_M)^{q-1} ,$$
$$\ln C_q(M) = -(q-1)\ln\delta_M + (q-1)\ln\Delta. \qquad (1.8)$$

The moments logarithm is in a linear dependence with the bin size logarithm. The random quantity with such behaviour is called an intermittent one and the factor of the logarithm of the bin size is called index of intermittence. The intermittent random quantity in a sense is the opposite of the Gaussian one, for which a considerable deviation from the average values is very improbable.

If even after averaging over all the events (events with both the same and different multiplicity can be averaged), the scaling relation

$$\ln\langle C_q(M)\rangle = -\lambda_q \ln\delta_M + g_q \ln\Delta \qquad (1.9)$$

is satisfied, then the physical process investigated is characterized by intermittence.

It is obvious that the experimental growth of normalized moments, revealed in a wide energy range of hadronic and leptonic collisions, is a new main characteristic of multiple production, which emphasizes the role of very-short-range correlations against the usual short-range ones responsible for resonance production.

The first phenomenological mechanism describing the behaviour of the factorial moments was the hypothesis of existence of two types of sources: laminary, with a regular signal distribution, and turbulent, which is characterized by chaotic bursts [7]. When colliding, the parton, passing through an interacting hadronic matter enters high-density regions (narrow channels), emits many particles, also passes through low-density regions (wide channels) and uniformly emits few particles. At such an interpretation, the main attention is drawn to the very complicated trajectory of the partons wandering in the hadronic matter [8]. But we believe, a much more natural way of interpretation of the anomalous behaviour of normalized moments is based on the hierarchy (self-similarity) of the processes of multiple production and on the closely connected with the self-similarity notion of fractal (multifractal) dimensionality.

Relations like (1.9) are a consequence of self-similarity in the structure studied, and give ground to carry out a dimensionality analysis. A dimensionality analysis means revealing in a 3N-dimensional momentum space (or in a one-dimensional rapidity space) lower-dimensional regions where the events are grouped.

At present there are available a number simulations of quark-gluon cascade development in hadronic matter [9,10]. The updating of the LUND program based on the realization of the idea of parton-hadron duality [11,12] led to realization of the fact that the unusual behaviour of normalized moments is due to the QCD cascade [12,13].

Before going on to the fractal analysis formalism, we shall show how a fractal (non-integer) dimensionality can arise in a simplest cascade process of decay of the massive particle $\mathfrak{m}$ [14] (see Fig.1).

On each self-similarity step of the cascade the mass decreases by a factor of $1/\alpha$, $\alpha\geq 2$ ($\alpha=2$, if final-state particles are produced with zero kinetic energy). On the r-th step of the cascade we have 2^r particles with mass $(1/\alpha)^r\mathfrak{m}$. The unification of masses of the particles obtained as a result of cascade, constitute the metric set $\varkappa$.

Let us show that at the beginning of the cascade process the topological dimension $d_T\varkappa=1$ and then, $d_T\varkappa<1$.

The topological dimension is equal to $\Im$, if it is possible to enter the finite open coverage of the multiplicity $\leq\Im+1$ into any finite open coverage of the set $\varkappa$, and if there exists such finite open coverages of $\varkappa$ into which it is possible to enter the finite open coverages of the multiplicity $<\Im+2$. The coverage multiplicity is the maximum number of coverage elements containing common points of the set $\varkappa$ [15]. For our example, the possibility of entering coverages of factor 2 into any open coverage of $\varkappa$ is a necessary condition for the dimension to be equal to unity. It is possible for 1a - it is enough to take somewhat shorter intervals of coverage and they also will intersect, i.e. the multiplicity is 2; and for 1b it is impossible, since the intersecting intervals cannot be embedded in the non-intersecting ones.

2. The Technique of Dimensionality Analysis

The cascade processes which are frequent in the high-energy physics, are due to some characteristic dimensionality. But, in contrast to the ideal self-similar cascades or geometric figures (e.g., Serpinski's carpet), in real physical systems there are possible deviations from self-similarity and, first of all, they contain not a single, but several characteristic scales connected with some dimensionality. The main idea of the dimensionality analysis is revealing these dimensionalities and trying to relate them with the dynamic mechanisms responsible for their production.

A strong mathematical definition of the topological dimensionality was made by the efforts of Freche, Hausdorff and Poincare in the beginning of the century. The capacity definitions of dimensionality were given later, which were then generalized to a non-integer case:

$$d_F = -\lim_{l\to 0} \ln N(l) / \ln (l) , \qquad (2.1)$$

where N(l) is the coverage of the set under investigation by open l-balls.

It can be shown that $d_F \le d_T$, and if $d_F < d_T$, then the object is called a fractal one, i.e. having a fractional dimensionality. Note that capacity has a purely geometric nature.

A set of events registered in an experiment fill the momentum space very non-uniformly, reflecting via its structure the dynamic mechanisms of particle production. That is why the events distribution over N(l) bins will be highly non-uniform and this non-uniformity with a physical meaning is not reflected by the capacity at all.

To generalize the notion of capacity, it is necessary to choose a universal measure fit to characterize the momentum space structure non-uniformities. The subject of measure was discussed in the problem of description of the dynamic systems turning to chaos [16]. For such systems, due to the necessity for transition from time averages to spatial ones, invariance of measure is required. There is no such problem for experimental data analysis, since the object (a population of points) can be considered as a given one and the time is not an essential characteristic. Besides, the object is a compact: for any open coverage there exists a finite subcoverage.

Let us consider the l-coverage of the compact. In each bir determine $N_i(l)$ probability (cellular) measure (mass):

$$p_i(l) = \int_{\Lambda} d\rho(x) \, , \qquad (2.2)$$

where Λ is the volume of a bin with l size, $\rho(x)$ is probability density function determined in the whole space by means of some non-parametric method, by the experimental data or by realization of the Monte Carlo simulation program [17].

From the point of view of the resolution of experimental installations, it is important to transit to the cellular measure $p_i(l)$, though l should not be arbitrarily small, so that the integral $\int_{\Lambda} d\rho(x)$ becomes senseless.

The basic approach to the dimensionality analysis lies in characterization of physical systems by the invariant probability measure singularities [18]. To do this, let us determine the scaling of the moments of the random quantity $p_i(l)$ of order q at scale l:

$$C_q(l) \equiv \langle p_i(l)^q \rangle \equiv \sum_{i=1}^{N(l)} p_i(l)^{q+1} \sim l^{\phi(q)} \, , \quad \phi(q) = q d_{q+1} \, , \qquad (2.3)$$

where d_q are the Renyi dimensions (generalized dimensions) determined for $\infty < q < \infty$. At $q=-1$, the relation (2.3) determines the capacity dimension $d_F = d_0$, at $q=0$ the information dimensionality d_1, and at $q=1$ the correlation dimension d_2.

If the fractal is uniform (geometric), then

$p_i = p = 1/N_l \quad N_l = N(l)$, and

$$(1/N_l)^{q+1} N_l \sim l^{qd_{q+1}}, \text{ hence we obtain for all } q: \qquad (2.4)$$

$$\ln N_l \sim -d_0 \ln \quad ,$$

i.e. for uniform fractals the Renyi dimensions of any order are the same and are equal to the fractal dimension, and the scaling of the q-th order momentum is characterized by the index qd_0, which linearly increases with the momentum order. And if the fractal is non-uniform, then all d_q are different (anomalous scaling) and the deviation from the dimensionality can be characterized by:

$$d_q - qd_0 \ . \qquad (2.5)$$

Thus, as in case of normalized moments (1.5), the Renyi dimensions can serve as quantitative power indices of non-uniformity of both the rapidity distribution and the hadron distribution in the momentum space.

The Renyi dimensions are defined as a slope connecting some values of $\{l_i\}$ with the corresponding values of $\{C_q(l_i)\}$ in a double-logarithmic scale.

But the direct application of the formula (2.3) to Renyi dimension calculation is rather time-consuming and what is more, there are no instructions regarding the choice of the box-size sequence $\{l_i\}$. The proposed algorithms based on nearest neighbour information (NN-algorithms) are much more efficient than the box-counting algorithms and they introduce a natural scale - the sample-averaged distance to NN, ... $k=1,2...M$, M is the total number of events in the studied ... (sample s...

Using the ergodic theorem one can make a replacement:

$$\sum_{i=1}^{N(l)} p_i(l)^{q+1} \sim \sum_{j=1}^{M} \tilde{p}_j \sim Q_l \qquad (2.6)$$

where $\tilde{p}_j$ is the probability to find the point of the studied set not in the box of size l but inside the hyperball of radius l, centered at some other point of the studied set and Q_l is the total number of q-tuples within these balls.

For the $\bar{R}_k$ sequence the scaling relation takes the form:

$$Q_{\bar{R}_k} \sim \bar{R}_k^{\phi(q)} \qquad (2.7)$$

For q=1 (correlation dimension) the number of q-tuples equals simply to the number of the sample events within l-balls and the left-hand side of (2.7) is equivalent to the mean number of the sample points being inside a hyperball with a radius equal to the average distance to the K-th neighbour, i.e. equals to the number k, so:

$$k \sim \bar{R}_k^{d_2} . \qquad (2.8)$$

Hence, the modified algorithm defines d_2 as a slope of the k-dependence of $\bar{R}_k$ in a double-logarithmic scale.

Fig.2 shows such a dependence used to define the correlation dimension of the Serpinski carpet. The dimension was determined by the least square method through 25 points: ln(of the number of the nearest neighbour 1,3...49)~ln(the sample-averaged distance to the nearest neighbour). Of course, the number of events must be large enough (there is a definite relation between the space dimensionality and the minimum number of events, by the use of which it is possible to draw consistent conclusions).

By the $\phi(q)$ dependence it is possible to classify different events of multiple production [21], as a multifractal object can be considered as an interwoven family of uniform fractals, each obeying the scaling law with index d_0^{α}.

Note that the dimensionalities of d_0^{α} are not in any way connected with the regions where singularities of the probability measure arise, i.e. it is impossible to recover the spatial structure of the multifractal support by the d_q spectrum. That is why we believe the local dimensionality introduced in Ref.[22] may be useful in separating the momentum space regions where considerable fluctuations of the invariant probability measure are observed.

Description of the algorithm for the local and global correlation dimension calculation is presented in the next section, and what is more, an interesting relation of the fractal dimensions to the "intrinsic dimension", a notion developed in the frame of the mathematical theory of pattern recognition, is also shown.

3. KNN Estimation of Probability Density. Local and Global Dimensionality.

Consider KNN estimation of probability density [23] which is a development of the well-known histogram method :

$$p_k(\mathbf{x}_i) = \frac{k}{M\, V_k(\mathbf{x}_i)} \tag{3.1}$$

where $V_k(\mathbf{x}_i)$ is the volume of a d-dimensional hypersphere containing the k nearest to $\mathbf{x}_i$ representatives of the set (sample) studied:

$$V_k(\mathbf{x}_i) = V_d R_k^d \; ; \; V_d = \frac{\pi^{d/2}}{\Gamma(d/2+1)} \; , \tag{3.2}$$

where R_k is the distance to the k-th nearest neighbour of $\mathbf{x}_i$, $\Gamma(z)$- is gamma function. From (3.1) and (3.2) we can readily obtain (see ref.[24]):

$$\ln R_k(\mathbf{x}_i) = \frac{1}{d} \ln K + \ln [M\, V_d p_k(\mathbf{x}_i)]^{-1/d}. \tag{3.3}$$

Eq.(3.3) cannot be solved relative to d, since the estimate of $p(\mathbf{x}_i)$, as one can see from (3.1), depends on K. Therefore, let us average R_k over the whole sample, according to the distribution function :

$$f_{k,x}(R) = C\, d\, R^{d-1} \frac{(CR^d)^{k-1}}{\Gamma(k)} \exp(-CR^d), \tag{3.4}$$

where $C = Mp(\mathbf{x})V_d$.

In the approximation of small R and large M we'll obtain the following equations :

$$\ln G_{k,d} + \ln \bar{R}_k = \frac{1}{d} \ln K + \text{const}, \qquad (3.5)$$

$$G_{k,d} = K^{1/d}\Gamma(k)/\Gamma(k + 1/d),$$

where $\bar{R}_k$ is the sample-averaged distance to the K-th nearest neighbour and "const" is independent of k.

The difference of this scaling equation from the previous ones obtained by a completely different approach consists in the so-called iterative addition $G_{k,d}$, which is close to zero for all k and d. Therefore, we solve this equation iteratively, first assuming $G_{k,d}=0$, and then, having obtained d_i, we calculate G_{k,d_i} and determine the value of d_{i+1}. We'll stop the iterations when d practically is no longer changed.

Such verification of d estimates is connected with averaging of the correlation integral. The correlation integral, the number of the sample points inside a hyperball of a fixed radius, is a random variable belonging to a binomial distribution with parameter $P(x)$ (the probability for the sample point to fall within this hyperball). Notice, our estimate is a global estimate, i.e. the whole sample is characterized by one number, though local differences are possible. From this point of view, local dimensionality is much more interesting, since we'll be able to detect local inhomogeneities corresponding to various dynamic mechanisms.

Consider eq.(3.3) again. Apart from sample averaging, there is also one more way to get a linear equation for dimension determination. For this, one must choose $\{k_j\}$ series such, that the density estimates are very close and hence, the dependence of $p_k(x)$ on k can be ignored. Following these chosen values $\{k_j\}$ and the corresponding $\{R_{k_j}(x_i)\}$, one can determine the estimate of the local dimension at a point **x**.

4. The Simulation Study

The Renyi dimension was determined for the samples generated by the algorithm for the Serpinski carpet (Fig.3), Henon map, and for samples obtained by different random number generators.

Experiments were carried out to investigate the method sensitivity to the choice of parameters which include: the sample size, sequence of the nearest neighbours, the order of the Renyi dimensions, and to study the possibilities of separation of the regions with anomalous structure. The important for many applications aspect of the quality of the quasi-random number generators was also considered. For comparison of the uniformity of the population of an N-dimensional space by "random" numbers, there were used "quasi-random" numbers - LP-sieves, which uniformly fill an N-dimensional cube [25].

Fig.4 presents the Renyi dimensions of order from 1 to 15 - the function $\phi(q)$. The three random-number generators being compared are: RNDM, which was widely used in the past decade; RANECU, a generator lately recommended by F.James [26] and NORIK, a matrix generator designed in the Yerevan Physics Institute [27].

Sets of two-dimensional random quantities distributed in a square of side 1 were considered. The slopes connecting the values of the moments of the invariant probability measure (2.3) were calculated through 70 points for distances equal to the average distance to the nearest neighbours with numbers from 6 to 75, the orders of dimensions being chosen from 1 to 15, the size of samples was 1000 and 5000.

For a strictly periodical structure of LP-sieves, all the Renyi dimensions are the same: $\phi(q)=qd_0$, the random number generators show some deviation from uniformity, which is due to limitedness of the sample. The matrix generator reveals somewhat better results.

Fig.5 presents Renyi dimensions calculated using different $\bar{R}_k$-sequences ($\bar{R}_k$ sequence consists of an average distances from 1 to 5, 1 to 25,..., 1 to 75 nearest neighbours). The smaller the range over which the dimension is determined, the more the random fluctuations and the more the difference between the function $\phi(q)$ and the line $y=qd_0$, which corresponds to complete uniformity.

Fig.6 shows the histogram of the local dimensions of a mixed sample consisting of a mixture of 500 events of Serpinski's carpet (d_2~1.9) and 500 events of Henon's map (d_2~1.2). Two peaks are clearly seen, which correspond to two modes (the correlation dimensionality is binned).

Unimodal distributions corresponding to data of the same type are shown in Figs.7 and 8.

A quasi-periodical distribution was used to "scan" the fractal support with the purpose to determine the anomalous areas: the dimensionality was calculated in the nodes of the LP-sieve (fig.9). Fig.10 presents the results of scanning of a square of side 0.9, where the Serpinski carpet is situated. For the sieve points fallen into the empty areas of the carpet the fractal dimension turned out to be >2.2, which allows them to be reliably separated.

The quasi-random sequence itself also turned out to be non-uniform on the boundaries of its support shown in Fig.11.

The program code is written in Fortran-77 for VAX- and IBM-type computers (operational system VM). Some subroutine from KNN multivariate density estimation package [17] are used for NN distances calculations and Q-tuples count. The calculations have been carried out on a EC-1046 computer in the computation center of the Yerevan Physics Institute.

CONCLUSION

To summarize, we have investigated a new method of multiparticle data handling, allowing to deal with the large amount of particles produced in modern colliders.

We have demonstrated how the Renyi dimensions can be used as a quantitative measure to outline possible inhomogeneities in a 3N-dimensional momentum space or in the rapidity (pseudorapidity) distributions.

We introduce a simple technique for Renyi dimensions calculation. A universal scale, a sample-averaged distance to NN, was offered. A q-tuples counting algorithm provides an evaluation of Renyi dimensions in a sizable range of q-values. KNN algorithm for correlation dimension calculation is much more suitable and precise compared to box-counting algorithms.

By the local dimension distribution obtained on fractal support we can judge about different mechanisms which took part in the creation of the given set.

The application of these ideas to the analysis of multiparticle production dynamics, requires intensive Monte Carlo simulations and detailed quantitative comparisons of simulated and experimental data.

I express my sincere gratitude to S.G.Matinyan for helpful discussions.

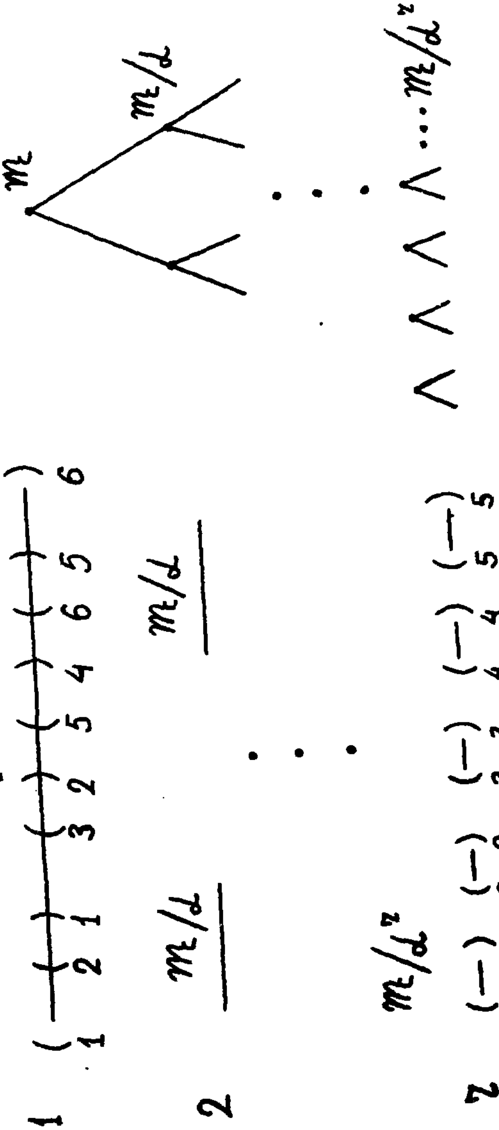

Fig.1

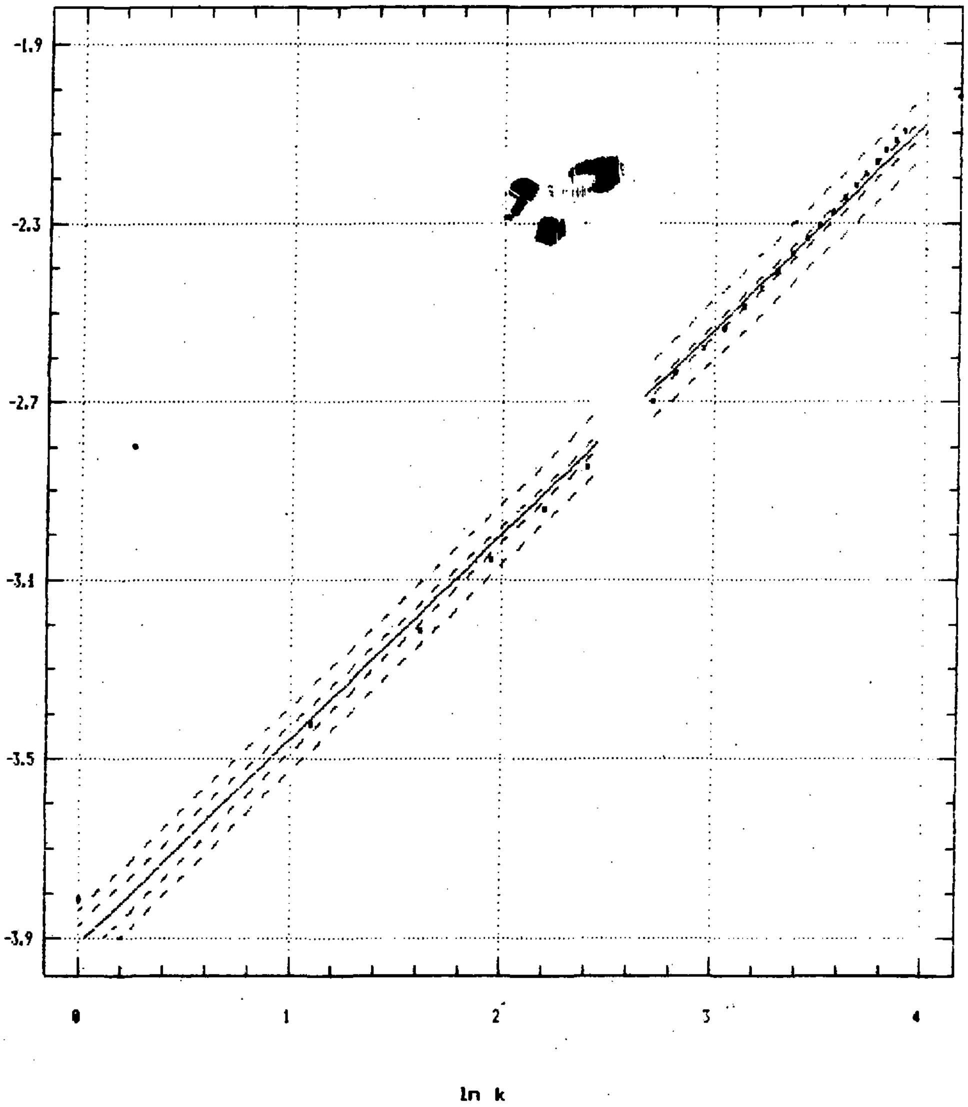

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Fig.3

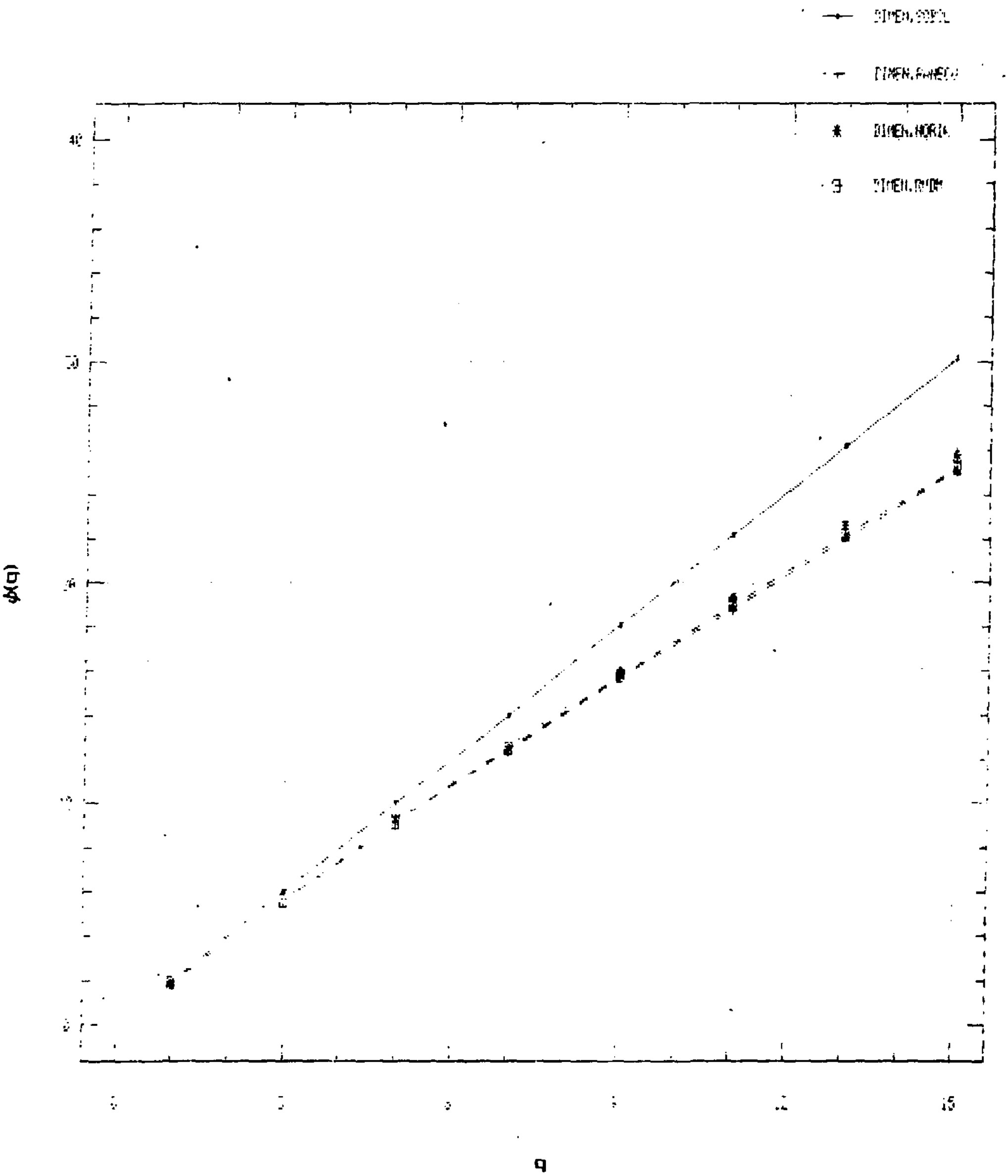
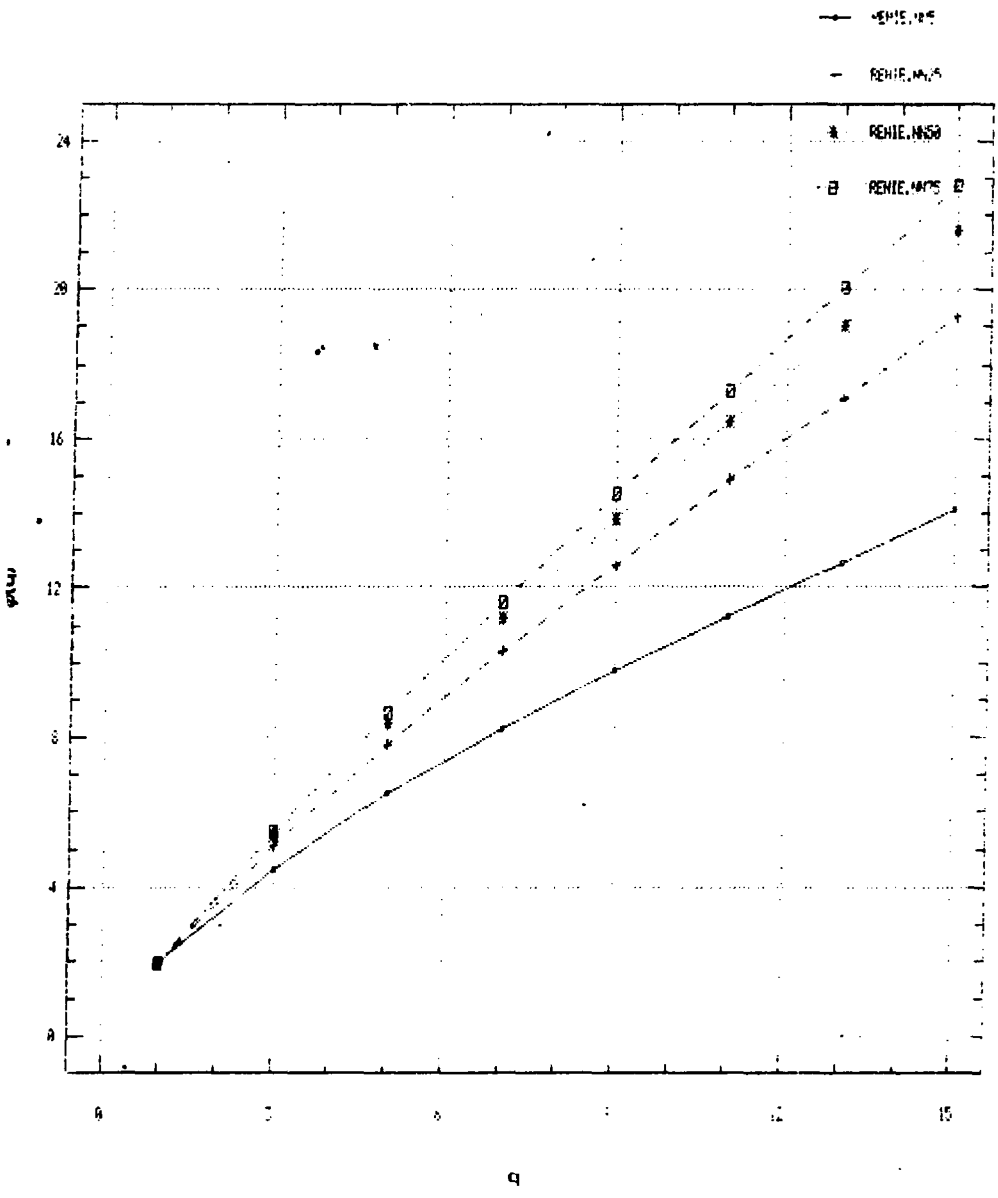


Fig.4



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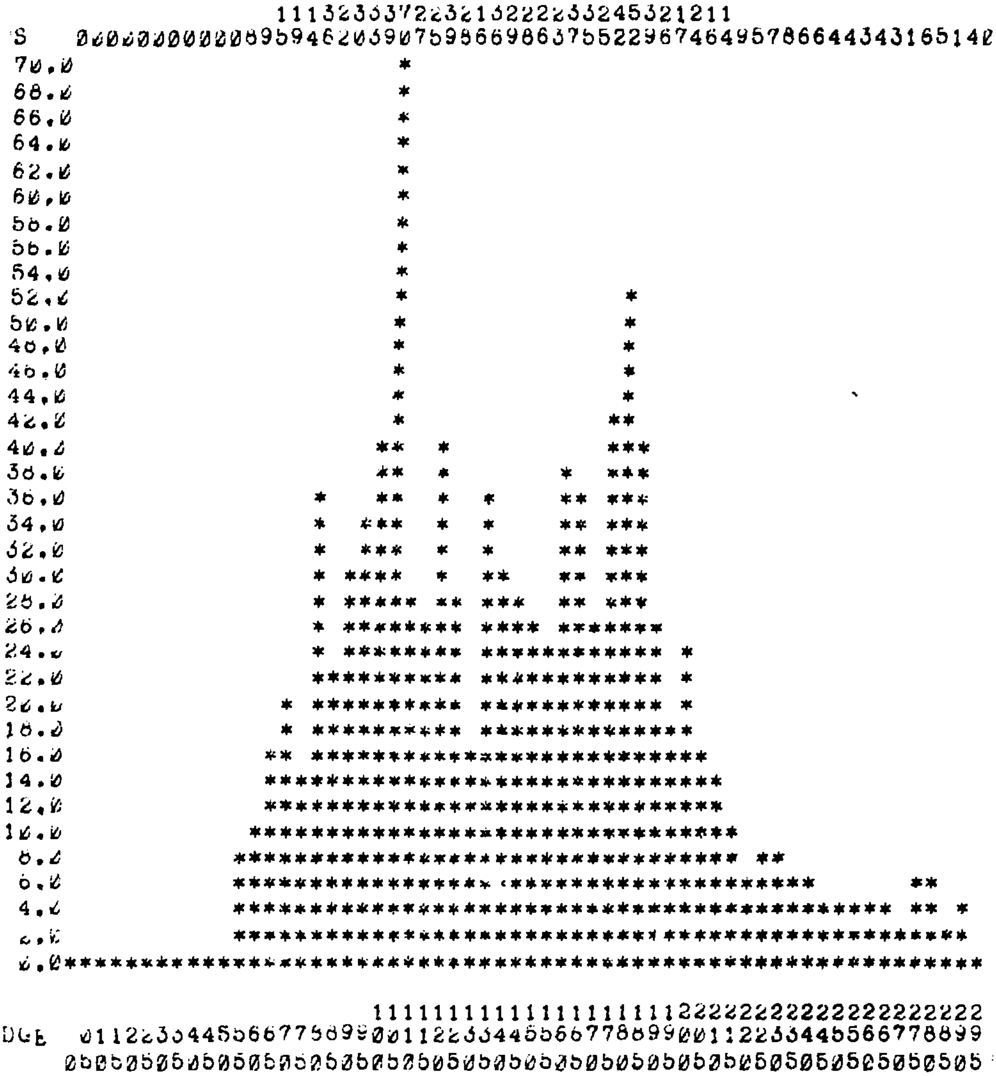
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Fig.6

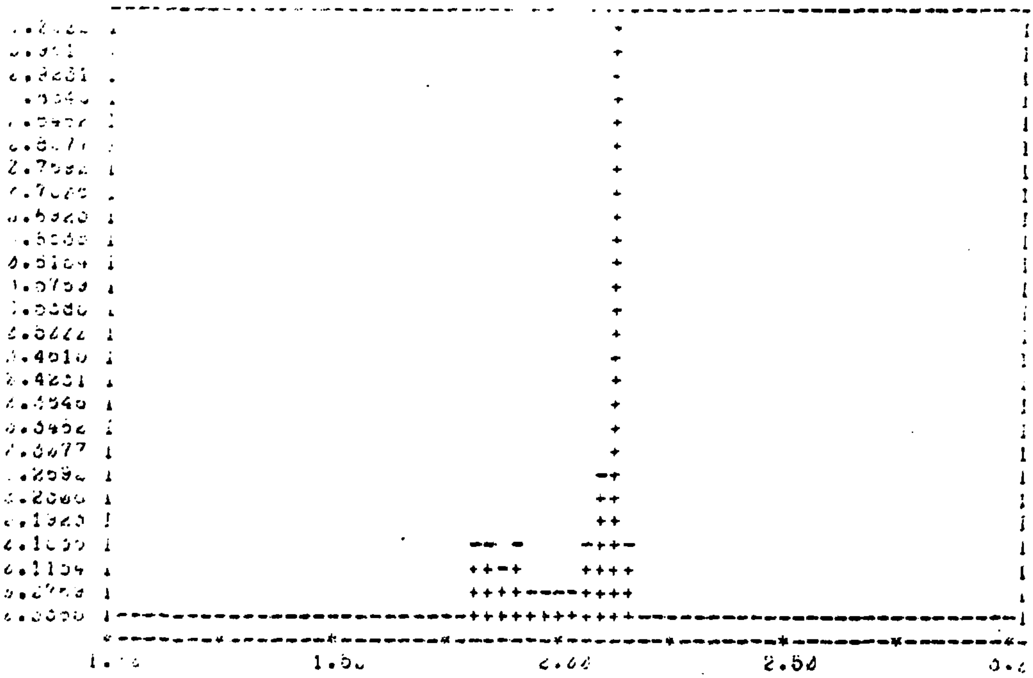

Fig.7

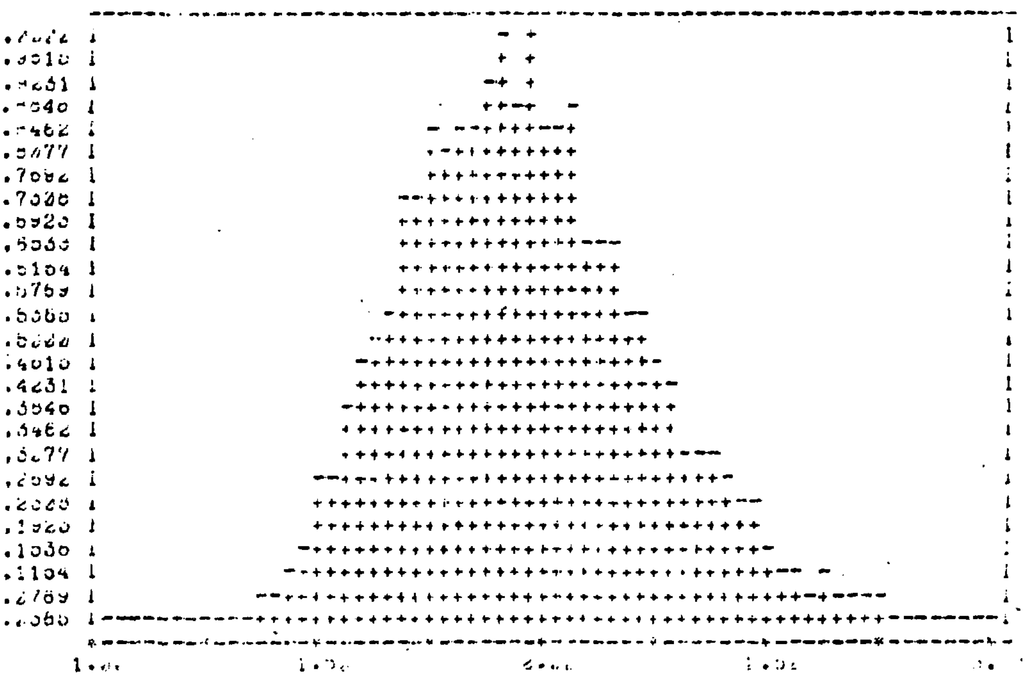

Fig.8

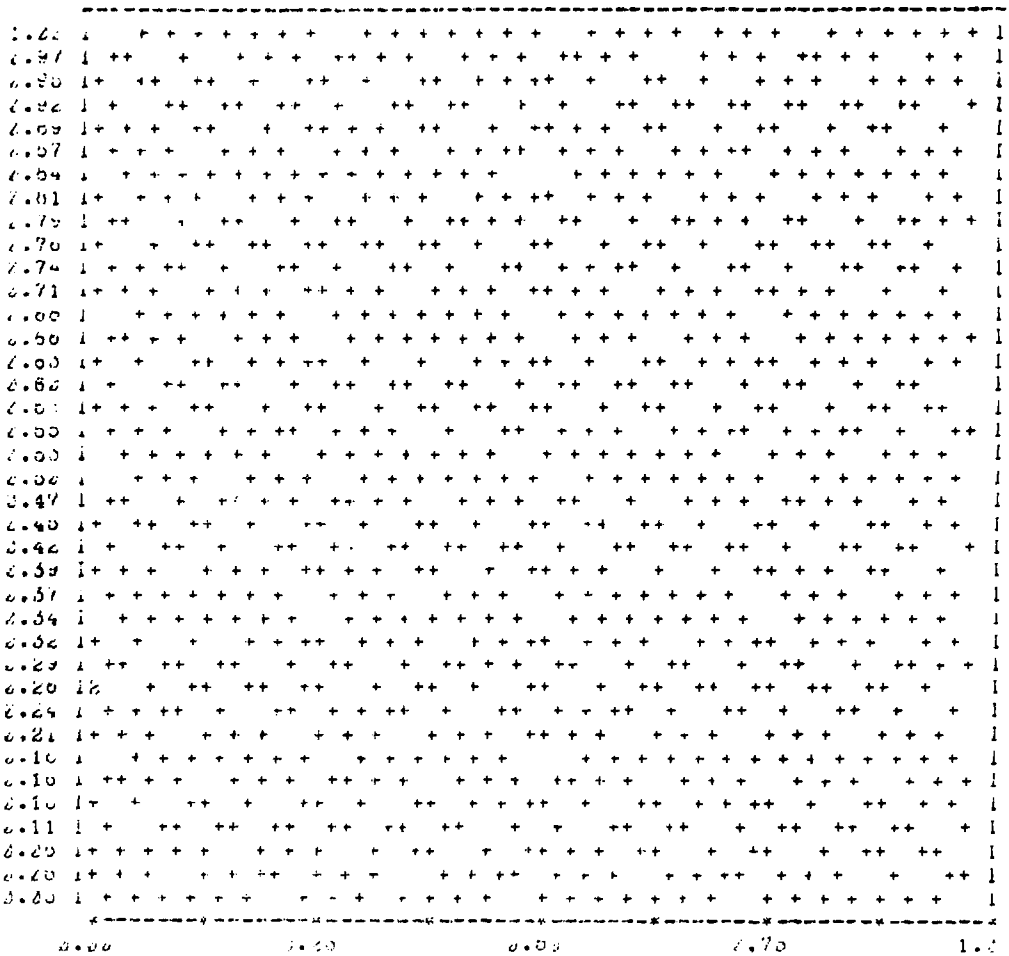

Fig.9

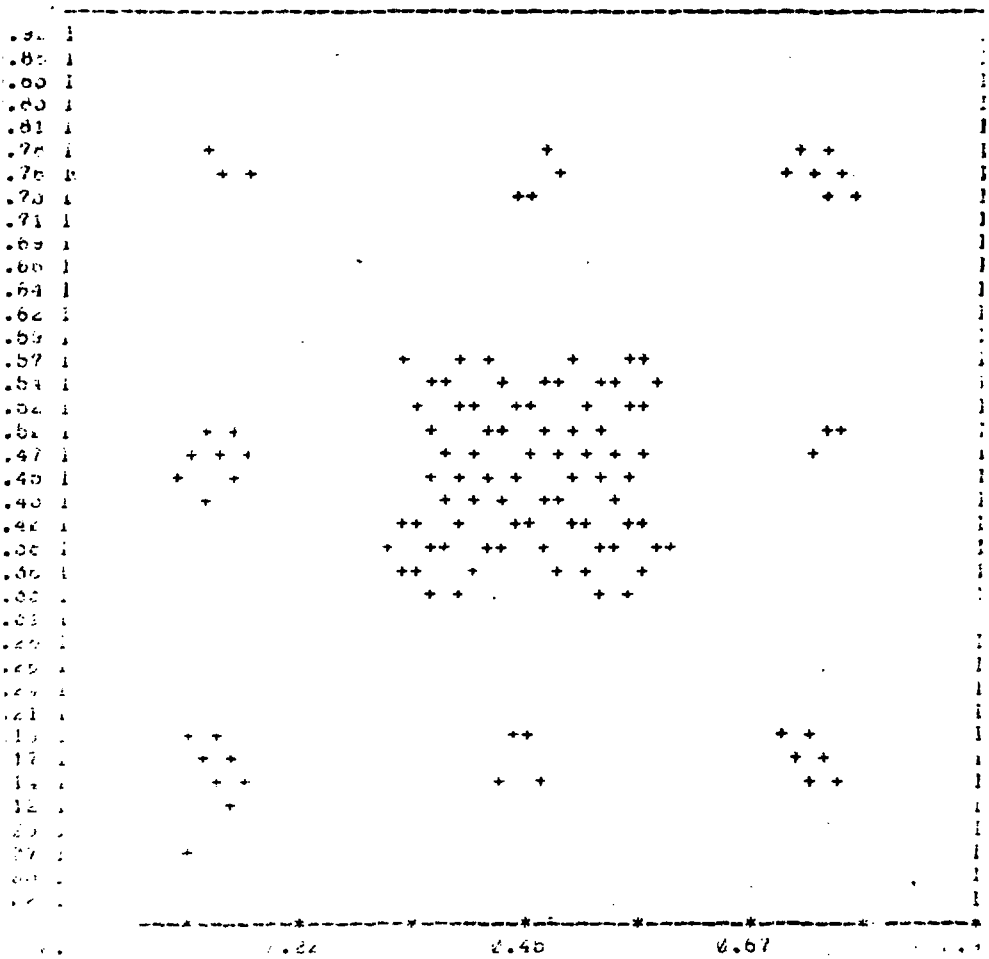

Fig.10

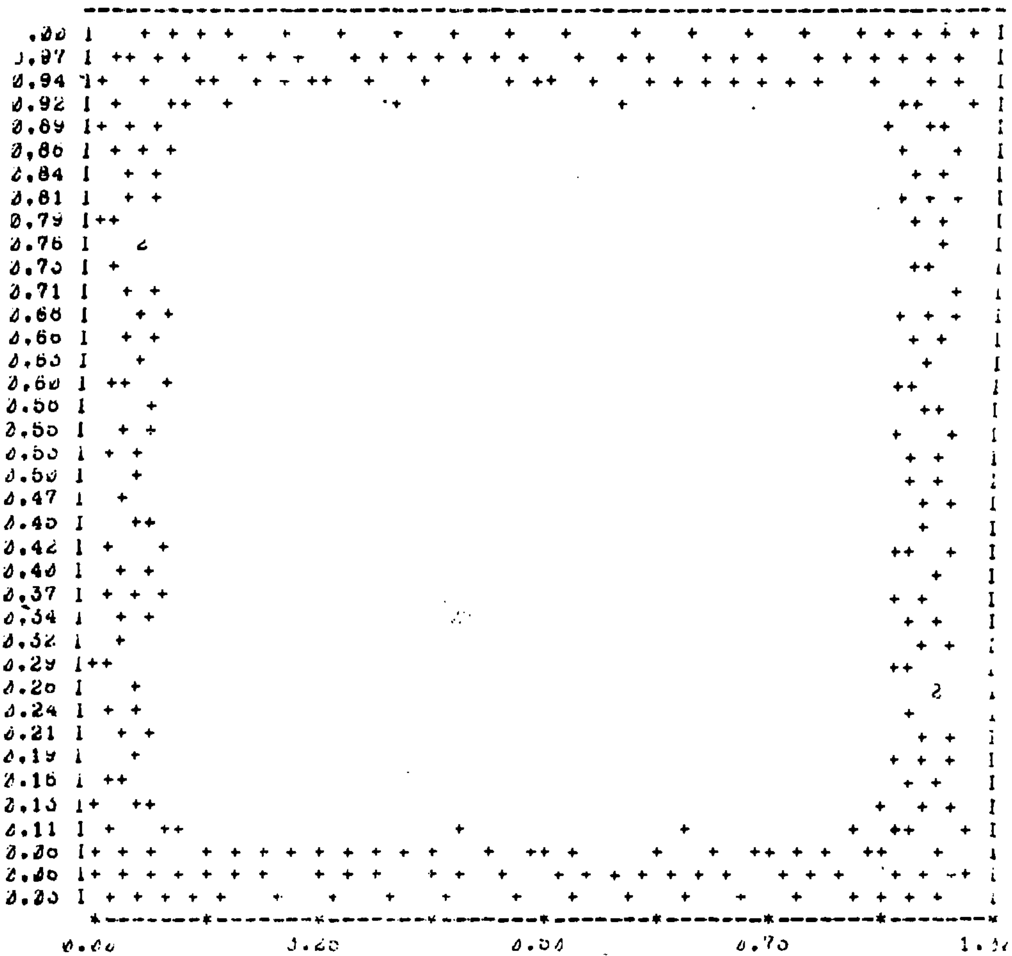

Fig.11

FIGURE CAPTIONS

Fig.1 Self-similar cascade decay of a particle with mass m. On the r-th step of the cascade development there are 2^r particles with mass m/α^r.

Fig.2 The straight line slope determination, by which the correlation dimensionality of the Serpinski carpet is determined.

Fig.3 The 5th generation of the Serpinski's carpet, 5000 points.

Fig.4 The $\phi(q)$ curve.
For a completely uniform set of quasi-random numbers in square of side 1, all the Renyi dimensions are the same, the pseudo-random numbers somewhat deviate from uniformity.

Fig.5 Comparison of the degree of non-uniformity of population of a unit square by two-dimensional random numbers (the RNDM generator). The narrower the $\bar{R}_k$ sequence for determination of the Renyi dimension, the higher the non-uniformity.

Fig.6 Histogram of the local dimensionality of a mixed sample - a Serpinski carpet & a Henon map.

Figs.7,8 Distributions (normalized histograms) of the local dimensionalities determined for samples of two-dimensional quasi- and pseudo-random numbers.

Fig.9 A planar LP-sieve, 1024 nodes.

Fig.10 The results of scanning of a Serpinski carpet: + denote points where the local dimension is >2.2.

Fig.11 The results of scanning of a planar LP-sieve over the boundaries of its support; + denote points where the local dimension is >2.2.

REFERENCES

1. Carruthers P., Shin C.C. Mutual Information and Forward-Backward Correlations in Multihadron Production, PRL, 1989, vol. 62, p.2073.
2. Carruthers P., Shin C.C. Correlations and Fluctuations in Hadron Multiplicity Distributions, Preprint LA-UR 83-1231, Los Alamos, 1983.
3. UA1 Collaboration. Bose-Einstein Correlations in $p\bar{p}$ Interactions at $\sqrt{s}$=0.2 to 0.9TeV, Preprint CERN-EP/89-71, 1989.
4. Barnet T.H, Dake S. et al. Extremely High Multiplicities in High-Energy Nucleus-Nucleus Collisions, PRL, 1983, vol.50, p.2062.
5. Bialas A., Pescansci R. Intermittence in Multiparticle Production at High Energy. Nucl. Phys. 1988, vol.B308, p.857.
6. Satz H. Intermittence and Critical Behaviour. Preprint CERN TH.5312/89, 1989.
7. Dias de Deus J. Intermittence Model for Rapidity Particle Density Fluctuations. Preprint CERN TH.4722/87, 1987.
8. Дремин И.М. Фрактальность в процессах множественного рождения. Письма в ЖЭТФ, 1987, т.45, с.505.
9. Lonnblad L., Petterson U. ARIADNE-2 a Monte Carlo for QCD Cascades in the Colour Dipole Formulation. Preprint LU TP-88-15, Lund, Sweden, 1988.
10. Marchesini G., Webber B. Herwig - Monte Carlo Program for Event Genera[illegible]. Phys., 1988, vol.B310, p.461.

11. Azimov Yu.Z., Dokshizer Yu.L., Chose V.A., Troyan S.I. Coherence Effects in QCD Jets. Phys.Lett., 1985, vol.165B, p.147.

12. Dahlqvist P., Andersen B., Gustaffson G. Intermittence and Multifractal Structures in QCD Cascades. Preprint LU TP-89-5, Lund, Sweden, 1989.

13. Fialkowski K., Wosiek B., Wosiek J. Intermittence and QCD Jets. Krakov University preprint TPJU-6/89, Krakov 1989.

14. Sarcevic I., Satz H. Self-Similar Multihadron Production at High Energies. Preprint BNL 43183, Brookhaven, 1989.

15. Александров П.С., Пасынков Б.А. Введение в теорию размерностей, М.: Наука, 1973.

16. Eckman J.P., Ruelle D. Ergodic Theory of Chaos and Strange Attractors. Rev.Mod.Phys., 1985, vol.57, p.617.

17. Chilingarian A.A. Statistical Decisions under Nonparametric a priori Information. Comp.Phys.Comm., 1989, vol.54, p.381.

18. Paladin C., Vulpiani A. Anomalous Scaling Laws in Multifractal Objects. Phys.Rep., 1987, vol.156, No.4.

19. K. Pawelzik, H.S.Shuster, Generalized dimensions and entropies from a measured time series, Phys.Rev.A, 35(1987) 481.

20. J.G.Caputo, P.Atten, Metric entropy : an experimental means for characterizing and quantifying chaos, Phys.Rev.A, 35(1987) p. 1311.

21 Dremin I.M. The Fractal Correlation Measure for Multiple Production. Mod.Phys.Lett., 1988, vol.3, p.1333.

22. Chilingarian A.A., Harutunyan S.Kh. On the Possibility of a Multidimensional Kinematic Information Analysis by Means of Nearest-Neighbour Estimation of Dimensionality. NIM, 1989, vol.A281, p.388.

23. R.A.Tapia, T.R. Thompson, Nonparametric Probability Density Estimation, (The John Hopkins University Press, Baltimore and London, 1978).
24. Pettis K.W., Baily T.A., Tain A.K., Dubes R.C. An Intrinsic Dimensionality Estimator from Nearest Neighbour Information. IEEE Trans. on Pattern Anal. and Machine Intelligence, 1979, PAMI1, p.25.
25. Соболь И.М. Точки, равномерно заполняющие многомерный куб. М.: Знание, 1985, Математика и кибернетика, No.2.
26. James F. A Review of Pseudo-Random Number Generators. Preprint CERN DD/82/22, 1988.
27. Akopov N.Z., Savvidy G.K., Ter-Harutyunian N.G. Matrix Generator of Pseudorandom Numbers. University of Minnesota, Preprint TPI-MINN-89/13P, 1989.

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А. А. ЧИЛИНГАРЯН

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The address for requests:
Information Department
Yerevan Physics Institute
Alikhanian Brothers 2,
Yrevan, 375036
Armenia, USSR

ИНДЕКС 3649

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ