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Advances in Space Research 43 (2009) 702–707

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

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Statistical study of the detection of solar protons of highest energies at 20 January 2005

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Received 12 December 2007; received in revised form 10 June 2008; accepted 5 October 2008

Abstract

On January 20, 2005, 7:02–7:04 UT the Aragats Multichannel Muon Monitor (AMMM) registered enhancement of the high energy secondary muon flux (energy threshold ~5 GeV). The enhancement, lasting 3 min, has statistical significance of $\sim 4\sigma$ and is related to the X7.1 flare seen by the GOES satellite and the ground level enhancement detected by the world-wide network of neutron monitors and by muon detectors. The most probable proton energy corresponding to the measured 5 GeV muon flux is within 23–30 GeV. Due to upmost importance of the detection of solar particles of highest energies in presented paper we perform detailed statistical analysis of the detected peak. The statistical technique introduced in the paper is also appropriate for the searches of sources of ultra-high energy cosmic rays.

Keywords: Solar cosmic rays; Neutron monitors; Ground level enhancements

1. Introduction

Measurements of the energy spectra of the solar cosmic rays (SCR) up to several tens of GeV will significantly enlarge the basic knowledge on the universal processes of particle acceleration at the Sun and in the Universe and will provide important information for the timely warnings on Space Weather severe conditions. Experimental investigation of the SCR of highest energies is a very difficult problem, requiring large surfaces of the particle detectors located at middle and low latitudes. Solar cosmic rays are electrons, protons and stripped nucleus accelerated in vicinity of Sun in flaring processes and by shock waves driven by the coronal mass ejections (CME).

Solar energetic particles (SEP) sometimes are energetic enough to generate cascades of particles in terrestrial atmosphere. Cascade particles can reach surface and enlarge count rates of particle monitors normally detecting rather stable flux of cascade particles generated by much more energetic galactic cosmic rays (GCR). Such abrupt count rate changes due to SCR are called ground level enhancements (GLE), encountered not more than 10 times during ~11 years of solar activity cycle.

On 20 January 2005, during the recovery phase of the Forbush decrease a long lasting X-ray burst occurred near the west limb of the Sun (helio-coordinates: 14 N, 67 W). The start of X7.1 solar flare was at 06:36 UT and maximum of the X-ray flux at 7:01 UT. The fastest (relative to X-ray start time) SEP/GLE event of 23-cycle (GLE No. 69) was detected by space-born and surface particle detectors few minutes after the flare onset. The start of GLE was placed at 6:48; the maximal amplitude of 5000% recorded by neutron monitor (NM) at the south pole is the largest increase recorded by neutron monitors ever.

Particle detectors of the Aragats Space-Environmental Center (ASEC, see Chilingarian et al., 2003, 2005) detected significant excess of count rates at 7:00–8:00 UT. From 7:02 to 7:04 UT, the Aragats Multichannel Muon Monitor (AMMM) detected a peak with significance $\sim 4\sigma$. It was the first time that we detected a significant enhancement of the >5 GeV muon flux. This short enhancement at 7:02–7:04

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doi:10.1016/j.asr.2008.10.005

exactly coincides in time with peaks from Tibet NM (Miyasaka et al., 2005), Tibet Solar Neutron Telescope (Zhu et al., 2005) and Baksan surface array (Karpov et al., 2005).

AMMM is located under 14 m of soil and concrete in the underground hall of former ANI experiment (see details in Chilingarian et al., 2005) and include 42 1 m^2 area and 5 cm thick plastic scintillators; the mean count rate of the detector was ~126,000 per minute and relative root mean square error of 1-min time series (RRMSE, usually used measure of particle monitor performance) equals to 0.28%. Therefore, rather large area of detector and corresponding high accuracy allows detection of additional flux due to very weak flux of highest energy SCR.

Flux of the muons with energies above 5 GeV detected by the AMMM as we can see from Fig. 1 (Zazyan, 2008), is generated by the primary protons with energies above 15 GeV. The energy distribution of the “parent” protons giving rise to the energetic muons depends on the power index of the primary proton energy spectra. Energy spectrum of protons, accelerated in Galaxy is very well fitted in wide energy range by power function with index $\gamma = -2.7$; the power index of energy spectra of “solar” protons varies from $\gamma = -4$ till $\gamma = -7$ and less for GeV energies. Our study of energy spectra of GLE No. 69 (Chilingarian and Reymers, 2007) estimates power index between −4 and −5 around 7:00 UT, 20 January 2005, therefore, most probable energy of primary protons, as we can see from Fig. 1, is between 23 and 30 GeV. To gain an insight into distribution of primary energies we calculate also 10% and 90% quartiles (energy regions containing lowest and greatest 10% of distribution) outlining the “improbable” energy regions. Our calculations prove that the most stable distribution parameter is the mode – energy value at biggest histogram bin. This value remains rather stable when we change simulation model.

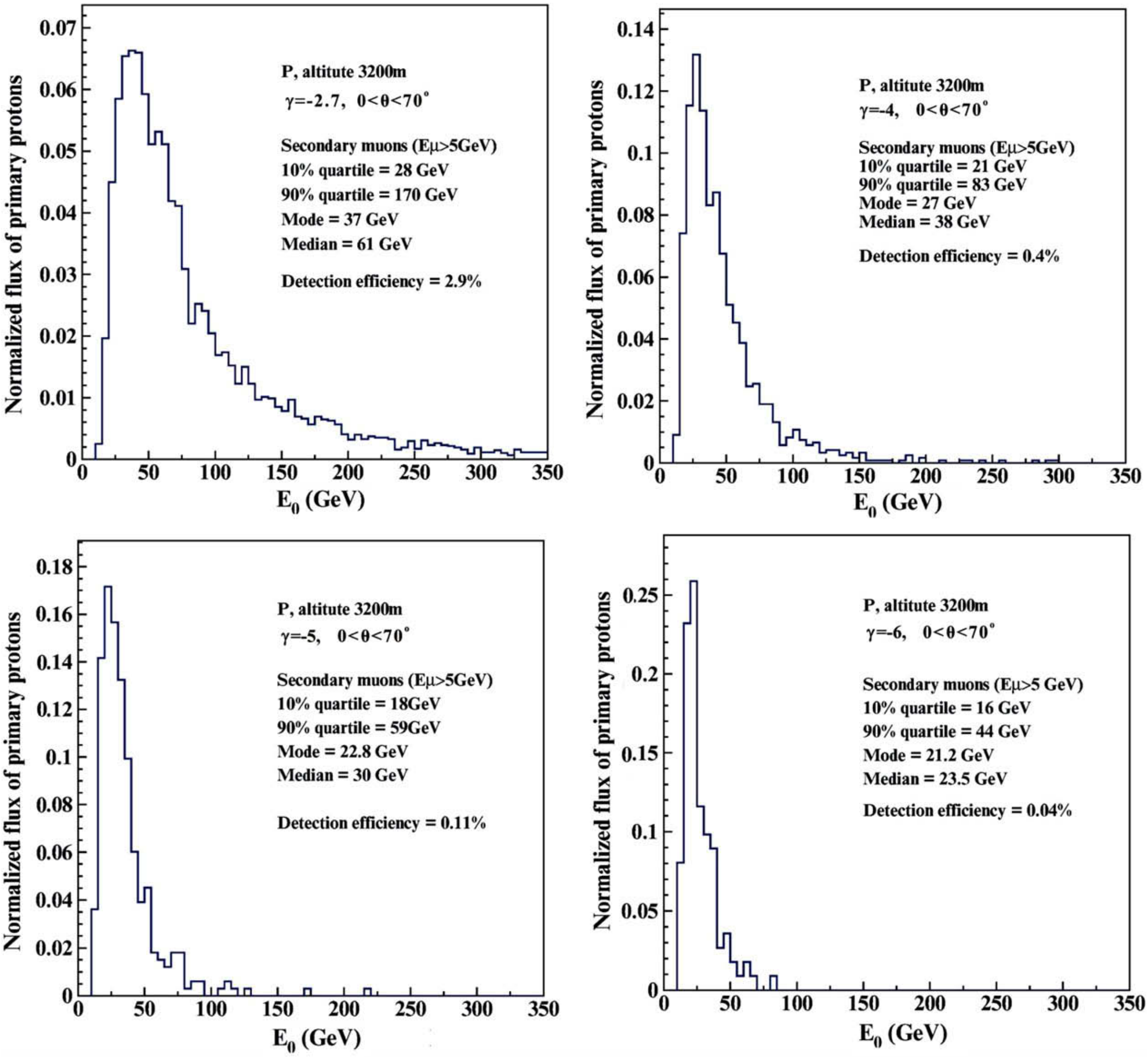

Fig. 1. Energy distribution of the primary protons that generate >5 GeV muons present at 3200 m altitude.

The details of AMMM detection of GLE No. 69, are discussed in Bostanjyan et al. (2007). In present paper due to upmost importance of the detection of solar particles with highest energies we discuss statistical methods used to reveal peak in the time series of AMMM. In the second section of paper we check the hypothesis that fluctuations of the count rates are well described by the Gaussian model, in the third section we introduce extreme statistical distribution, in forth we describe the computational experiment for checking obtained results.

2. Analysis of the residuals (checking the Gaussian model)

The difficulty of testing hypothesis of Gaussian nature lies in the slow drift of the mean count rate of time series due to systematic changes of several geophysical and interplanetary parameters. Disturbances of the interplanetary magnetic field (IMF) in the end of January 2005 (triggered by passage of several Interplanetary CMEs at 16–20 January) modulate cosmic ray flux, introducing trend in the secondary cosmic ray fluxes.

To account for the changing mean of the greater than 5 GeV muon flux we calculate the hourly mean count rates and corresponding residuals (fitting errors, differences between observed hourly means and values of 3-min count rates in this hour; 20 numbers for each hour):

$$X_{i,j} = \frac{C_{i,j} - \bar{C}_j}{S_j}, \qquad i = 1, 20 \quad j = 1, N_h \tag{1}$$

where $X_{i,j}$ are normalized residuals, $C_{i,j}$ are 3 min count rates of the AMMM at jth hour, $\bar{C}_j$ are hourly means of the 3-min time series and

$$S_j \approx \sqrt{\bar{C}_j}, \quad j = 1, N_h \tag{2}$$

are proxies of root mean square errors and N_h is number of hours.

Statistical distribution (1) represents, so called, multinomial process. Multinomial process consists of sum of j Gaussian random processes; in our case – time series of count rates corresponding to Gaussian process with same variance and different means. In our probabilistic treatment of the problem we normalize time series by the "moving" means $\bar{C}_j$ and variances S_j^2, estimated each hour. In this way we plan to obtain a proxy of the standard Gaussian distribution $N(0,1)$ to use later on as a test statistics.

To check our assumptions on Gaussian nature of the distribution (1) we perform calculation of residuals for 20 January 2005 and for whole January 2005. As we describe in Bostanjyan et al. (2007) we prepare 3-min time series from the 1 min ones. Joining 1 min time series in 3, 5, 10 or 60 min time series is ordinary operation used by the all groups running the neutron and muon monitors. To account for the arbitrary choice of the start minute we integrate other all three possibilities of different starts of the 3-min time series, therefore number of events in histograms is three times more than number of 3-min count rates.

The resulting histograms of the normalized residuals are shown in Figs. 2 and 3. We see rather good agreement with standard normal distribution $N(0,1)$; values of the χ^2 test are ~1 for degree of freedom. The maximal values of 3.77[1] (see the right tail of histogram in Fig. 2) corresponds to a peak at 7:02–7:04 UT. The same maximal value remains maximal also for the 1-month histogram (Fig. 3). The second maximal value for a month histogram is 3.64.

Proceeding from good agreement of histogram with Gaussian curve and from rather large value of the biggest residual, we can accept the hypothesis that there is additional signal superimposed on the galactic cosmic ray background. Of course, within validity of the Gaussian hypothesis this and larger values can encounter by chance, therefore we'll need additional statistical tests proving that detected peak is caused by the highest energy solar protons.

3. Calculation of the chance probability

As usually in statistical hypothesis testing, the hypothesis we want to check (named H_0) consists in the opposition to the hypotheses we are interested, i.e., we will check the hypothesis that there is no additional muons in 3-min time series ("no-signal" hypothesis) and, therefore, that detected peak is random fluctuation only. To prove the existence of signal, we have to reject H_0 with the maximal possible confidence. Detecting large deviations from H_0, i.e., very low probability of H_0 being true, do not imply that the opposite hypothesis is automatically valid. As was mentioned by Astone and D' Agostini (1999) behind logic of standard hypothesis testing is hidden a revised version of the classical proof by contradiction. "In standard dialectics, one assumes a hypothesis to be true, then looks for a logical consequence which is manifestly false, in order to reject the hypothesis. The "slight" difference introduced in "classical" statistical tests is that the false consequence is replaced by an improbable one".

If the experimental data will not differ significantly from test distribution obtained under assumption of "no-signal" hypothesis there will be no reason to reject H0 and therefore we cannot claim that AMMM detected high energy muons of "solar origin". And if we will be able to reject H_0, we can accept with definite level of confidence that there are high energy protons coming from the sun. Usually confidence level is enumerated as "chance probability", the probability of H_0 hypothesis to be true.

The statistical test for accepting or rejecting hypothesis is based on the maximal deviation from most probable value (3.77 in our case) observed in time series. The probability to obtain this or another maximal deviation depends on the number of events considered, i.e., on the time series

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[1] We obtain maximal value 3.77 instead of, reported in Bostanjyan et al. (2007), 3.93 due to slightly different procedure of residual calculation.

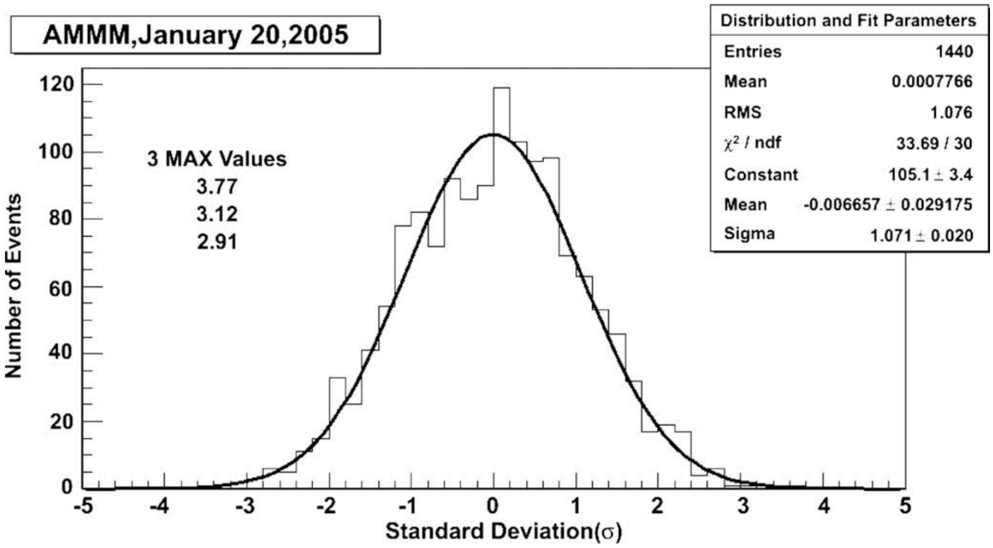

Fig. 2. Normalized residuals calculated by 3-min AMMM time series at 20 January 2005. In the picture legend are posted the histogram mean and RMS and also fitted curve mean and variance, as well as number of degrees of freedom in the χ^2 test.

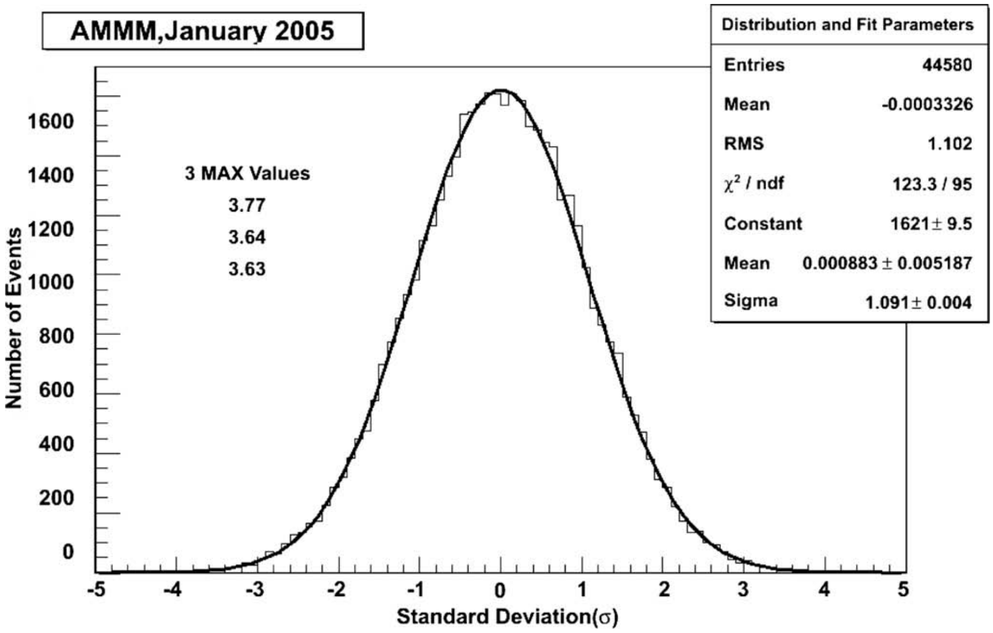

Fig. 3. Normalized residuals calculated by 3-min AMMM time series at January 2005. In the picture legend are posted the histogram mean and RMS and also fitted curve mean and variance, as well as number of degrees of freedom in the χ^2 test.

length. Therefore, the most appropriate test provides the extreme statistics distribution (Chapman et al., 2002; Chilingarian et al., 2006):

$$c^M(x) = M \cdot g(x)(1 - G_{>x})^{M-1} \tag{3}$$

where g(x) is standard Gaussian probability density $N(0,1)$.

$$G_{>x} = \int_x^{\infty} g(t)dt \tag{4}$$

is, so called, standard Gaussian distribution's *p*-value: the probability to obtain the value of test statistics greater than $\boldsymbol{x}$; M is number of attempts we made to find the biggest deviation from H_0 (number of elements of considered time-series multiplied by number of attempts we made to find greatest deviation).

To obtain the probability to observe extremely deviation equal to $\boldsymbol{x}$ among M identically distributed random variables (*p*-value of the distribution $c^M(x)$) we have to integrate $c^M(x)$ in the interval $[x, +\infty)$:

$$C_x^M = \int_x^{\infty} c^M(t)dt \tag{5}$$

$C_x{}^M\,(x)$, *p*-value of the distribution (3), equals to probability that observed test statistics $\boldsymbol{x}$ maximally deviates from the most probable value under assumption that H_0 is valid. And if this probability is low enough we can reject H_0 and accept alternative hypothesis that observed deviation is not fluctuation, but a contamination of the distribution of different statistical nature, i.e., a signal.

The probability to observe in one from 480 (i.e., during the day) of 3-min time-series count rate enhancement of 3.77 equals according to Eqs. (3) and (5) to:

$$C_{3.77}^{1440} = 0.1045$$

It means that in absence of any signal when examining daily variations of the 3-min count rates in one case from 10 it is expected to detect the deviation of the mean value equal to 3.77. Equivalent statement: approximately once in 10 days only we will detect 3.77 enhancements in the 3-min time series of AMMM.

However, we have to correlate the expected signal from protons, accelerated at Sun with time of X-ray flare and CME launch. Of course, we cannot expect the signal from solar protons before X-ray flare and an hour after the X-ray flare or/and CME launch occurs. The chance probability to detect a deviation equals to 3.77 in 1 h equals to $C_{3.77}^{60} = 0.0049$, i.e., only once in 200 cases we can expect such enhancement.

As we can see in Fig. 3 the second maximal monthly deviation equals to 3.64. If we accept hypothesis that 3.77 value was due to solar protons, we have to check if 3.64 is typical monthly maximal deviation. Calculated according (3)–(5) value of $C_{3.64}^{14340} = 0.2768$ is rather large and we have no reasons to reject H_0; i.e., at January 2005 the residual distribution (Fig. 3) was Gaussian with only one outlier attributed to high energy solar protons.

4. Effect of the multiple attempts in searches of “biggest deviation from H_0

To check assumption that when calculated significance of signal we should take into account three possible starts of time series we perform simulations with simple model of time series.

The model can be described as following:

(i) generate 1440 numbers from the standard normal distribution $N(0, 1)$;

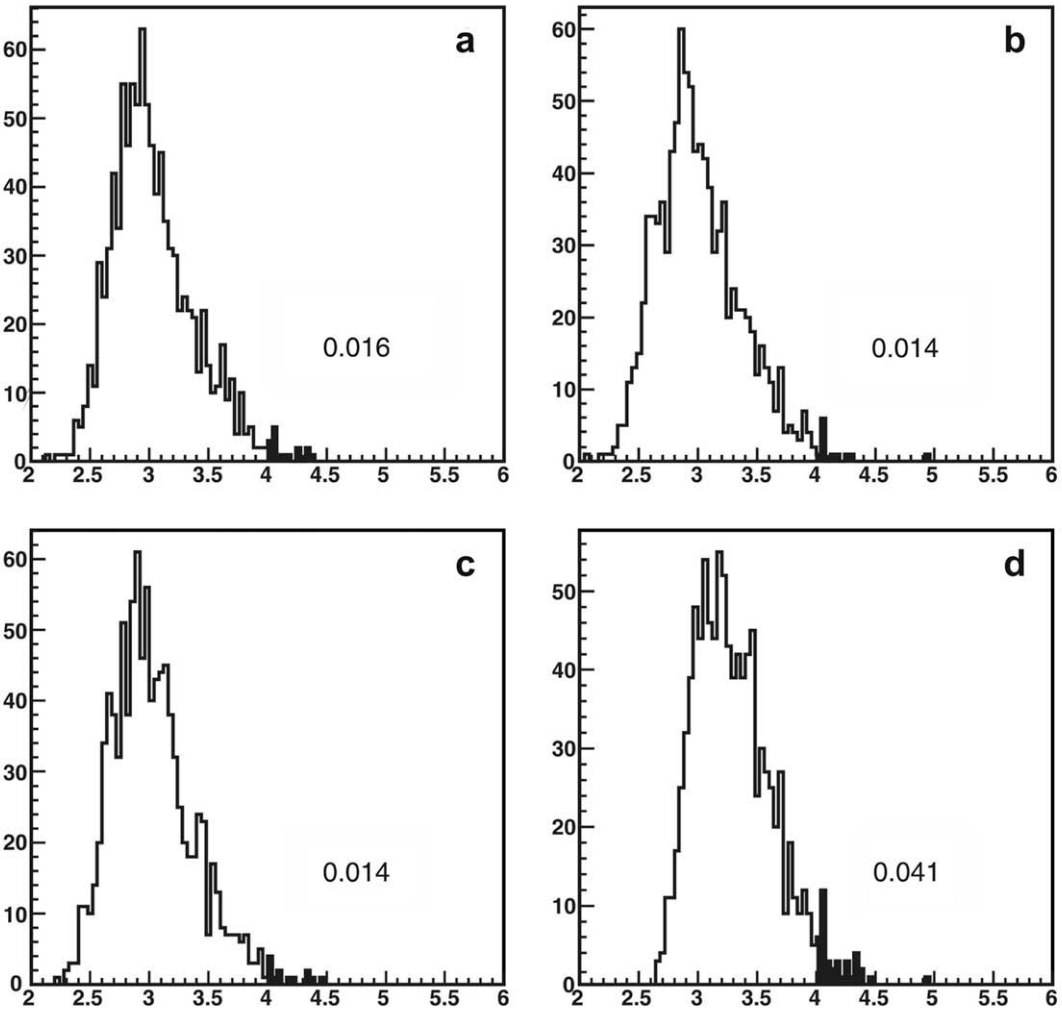

Fig. 4. Histograms of the extreme statistics. (a–c) selecting extreme statistics for three independent time series (iv); and (d) selecting maximal value among three extreme statistics – (v). Black area in the histograms denotes the summation region and number the integral (sum) value from 4 till infinity.

(ii) prepare three time series summing three consequent numbers of the raw, starting from the first, second and the third elements;
(iii) perform according to Eq. (1) normalization procedure (subtract the mean and divide to root of variance);
(iv) determine and store the maximal element of each of three normalized time series;
(v) determine and store the maximal element among three time-series maximums;
(vi) repeat (i)–(vi) 1000 times and consider four histograms of extreme values;
(vii) calculate the frequencies of obtaining values equal or greater than 4 (for simplicity we take 4 instead of 3.77).

Intuitively, when having three possibilities physicist will choose one that emphasis the presence of signal (the situation (v)). But as we can see from Fig. 4d, the probability to obtain the fake signal is dramatically enhanced (approximately by three times). From the same picture we can see that obtained in (d) chance probability 0.041 is in good agreement with value calculated according to Eqs. (3) and (5): $C_4^{1440} = 0.0436$.

5. Conclusion

On January 20, 2005, 7:02–7:04 UT the Aragats Multichannel Muon Monitor registered enhancement of the high energy secondary muon flux. The enhancement, lasting 3 min, has statistical significance of $\sim 4\sigma$ and chance probability – less than 0.5%.

Proposed statistical methodology of signal significance estimation can be recommended for the treatment of GLE events, especially for revealing weak signals of solar cosmic rays of the highest energies. The extreme statistics are useful tool for the enumeration of the significance of detected peaks in time series. When making different attempts to reject H_0 the probability of obtaining "fake" signal during a given time period increases approximately proportional to number of attempts.

Acknowledgments

Author is grateful to N. Bostanjyan, G. Karapetyan, A. Hovhannisyan, A. Reymers and Zazyan Meri for useful discussions and fruitful collaboration. Work was partly supported by the INTAS Grant 8777.

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