Primary energy estimation by EAS size and shape parameters for the MAKET and KASCADE

1 Introduction

The EAS data from MAKET (3200 m asl) and KASCADE (110 m asl) experiments are classified into light and heavy groups of nuclei and the primary energy of these nuclei are estimated by the Neural Net classification and estimation method. The same EAS parameters from both experiments are used to reconstruct the type and the energy of primari Cosmic Rays. These are the shower size - N_e and the shower shape parameter - *age*. Simulations of EAS for both experiments were performed using CORSIKA code with QGSJet model for high energy strong interactions. For KASCADE experiment the low energy interactions were simulated by EGS code, while for the MAKET experiment NKG approximation of electromagnetic cascades was used. The detailed detector response for both installations was taken into account. The energy range of simulated events in both cases is $10^{14} - 10^{17}$ eV. Showers initiated by five types of nuclei were generated: H, He, O, Si, Fe. The zenith angle of incidence of primary CR is varying form 0 to 45°. So the θ is also used as an input for Neural Net analysis.

The number of events used for the training of NN is ~ 35000 light and ~ 17500 heavy, and ~ 58000 light and ~ 34000 heavy for the MAKET and KASCADE experiments respectively. The comparative study of results of the mass classification and energy estimation for the MAKET and KASCADE experiments presented in the next section is based on the classification and estimation of independent control (not used for the NN training) sample.

2 Correlation matrices and one-dimensional statistical tests

Table 1: One-dimensional tests (t- Student, D- Kolmogorov-Smirnov, U- Mann-Whitney

	KASCADE			MAKET		
	t	D	U	t	D	U
lnN_e	52.60	20.68	45.50	56.40	22.97	51.54
S	44.38	20.22	44.31	70.28	27.90	63.48

Table 2: Correlation matrix for light sample (MAKET)

	lnE_0	lnN_e	S
lnE_0	1.00	0.99	-0.22
lnN_e	0.99	1.00	-0.31
S	-0.22	-0.31	1.00

Table 3: Correlation matrix for light sample (KASCADE)

	lnE_0	lnN_e	lnN_μ^{tr}	S
lnE_0	1.00	0.96	-0.98	-0.06
lnN_e	0.96	1.00	-0.97	-0.20
lnN_μ^{tr}	0.98	0.97	1.00	-0.11
S	-0.06	-0.20	-0.11	1.00

Table 4: Correlation matrix for heavy sample (MAKET)

	lnE_0	lnN_e	S
lnE_0	1.00	0.98	-0.33
lnN_e	0.98	1.00	-0.39
S	-0.33	-0.39	1.00

Table 5: Correlation matrix for heavy sample (KASCADE)

	lnE_0	lnN_e	lnN_μ^{tr}	S
lnE_0	1.00	0.96	0.99	-0.41
lnN_e	0.96	1.00	0.98	-0.46
lnN_μ^{tr}	0.99	0.98	1.00	-0.43
S	-0.41	-0.46	-0.43	1.00

3 Neural classification of EAS samples for MAKET and KASCADE into light and heavy groups

Each EAS event was first classified as induced by the light (H+He) or heavy (Si+Fe) groups of nuclei. The EAS events induced by intermediate O nuclei were not included in the training samples, as we need more or less etalon "pure" cases. As one can see from Table 6 the discriminative power of N_e and *age* parameters is higher on higher altitudes and the mean misclassifications are $\sim 5-10\%$ lower for MAKET samples as compared to KASCADE.

This result can be explained by the degradation of the cascade on see level leading to larger fluctuations of reconstructed EAS parameters and on almost complete overlapping of the two-dimensional distributions as one can see from Figures 1, 2. From these figures one can see broader distributions of both shower parameters and hence, larger overlap of H and Fe induced shower for KASCADE level, which makes the separation between light and heavy nuclei induced EAS very difficult. These findings are supported also by the results of one dimensional statistical tests for light and heavy samples. analysis. From Table 1, which shows different test values for MAKET and KASCADE light and heavy samples, one can see larger differences of N_e and *age* distributions for MAKET light and heavy samples, especially the shower age parameter shows a pronounced mass sensitivity for MAKET samples.

Table 6: Neural classification into two classes using H+He and Si+Fe events for MAKET and KASCADE

	MAKET		KASCADE	
	light	heavy	light	heavy
light	0.720	0.280	0.688	0.312
heavy	0.240	0.760	0.338	0.662

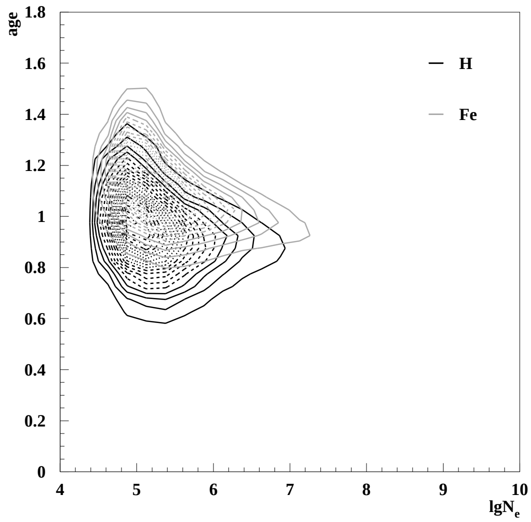

Figure 1: *The shower age versus shower size for proton and iron for the MAKET MC data*

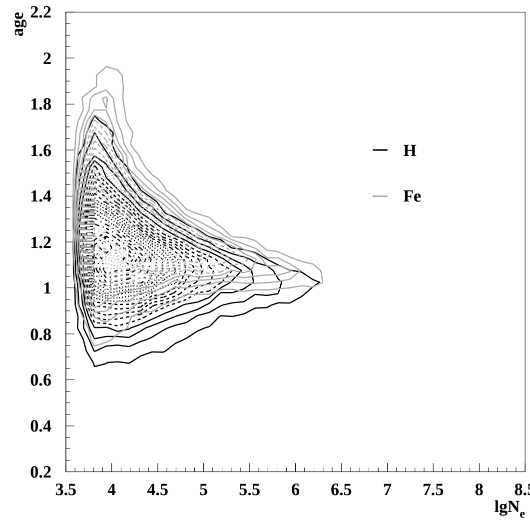

Figure 2: *The shower age versus shower size for proton and iron for the KASCADE MC data*

4 Estimation of the primary energy of different groups of nuclei

Primary energy of each EAS event is estimated separately for the light and heavy nuclei induced events. Figures 3, 5, 4 and 6 show the realtive error of the energy estimation versus the primary energy for light and heavy groups of nuclei for MAKET and KASCADE respectively. It is easy to see that the energy estimation for the light group of nuclei is almost unbiased for both MAKET and KASCADE data over the energy range $3 \times 10^{14} - 3 \times 10^{16}$ eV. For the heavy nuclei one observes a small bias (less than 10%) of energy estimation in both cases. For both groups of nuclei the spread of the energy estimation is significantly larger for KASCADE data, which is again caused by stronger fluctuations of shower parameters on see level. Figures 7, 8 9, 10, 11, 12, 13 and 14 show the distribution of Ne parameter for the narrow energy bins for light and heavy nuclei for both experiments. From these figures one can see that the width (deviation) of the N_e distribution for fixed primary energy is always larger for KASCADE events. The same can be observed for the shower *age* distribution (Figures 15, 16, 17, 18,19, 20,21, 22). Larger overlap of *age* distributions yields also to worse separation between light and heavy primaries.

Tables 2, 3, 4 and 5 demonstrate the parameter correlations in light and heavy groups for MAKET and KASCADE samples respectively. From these tables one can see very strong correlation of N_e parameter with primary energy for both, light and heavy primaries in MAKET samples. For the KASCADE data these correlations are slightly less, demonstrating again that N_e is better energy estimatior at higher altitudes. The shower age parameter is much less sensitive to the primary energy of CR, forthermore, for the light primaries at KASCADE level *age* shows negligible correlation with energy.

So, using only EAS size and shape parameters for classification and estimation tasks one obtaines significantly better results for higher altitudes (near the shower development maximum) as compared to see level due to the less fluctuations of EAS parameters.

One should also consider the possibility of using another EAS parameter for the classification and estimation tasks. This is the EAS muon size - N_μ parameter which is available in KASCADE data. From Tables 3 and 5 one observes that this parameter has very strong correlation with primary energy and is as good energy estimator as N_e parameter in case of MAKET data. Including this additional parameter in data analysis yileds to significant improvement in primary energy estimation for both, light and heavy nuclei. Figures 23 and 24 demonstrate that the bias of energy estimation became smaller for heavy primaries and the relative error of estimation is dramatically decreased for light and heavy primaries. The results of classification into light and heavy primaries are also significantly improved. Table 7 shows that misclassifcations are decreased from 31 to 19% for the light and from 34 to 16% for the heavy primaries respectively.

Unfortunately this EAS parameter is not available for MAKET data. One could expect to improve significantly the classification and estimation results using N_μ parameter for MAKET data analysis, and even to classify EAS data as induced by 3 or more types of primary particles and to estimate their primary energy with high accuracy.

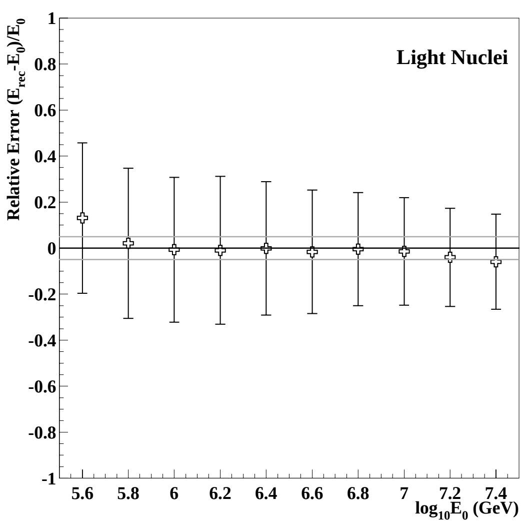

Figure 3: *Primary energy estimation of light nuclei (H,He) (MAKET)*

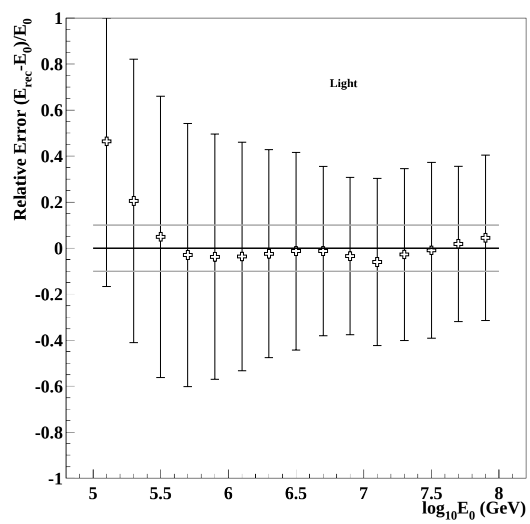

Figure 4: *Primary energy estimation of light nuclei (H,He) (KASCADE)*

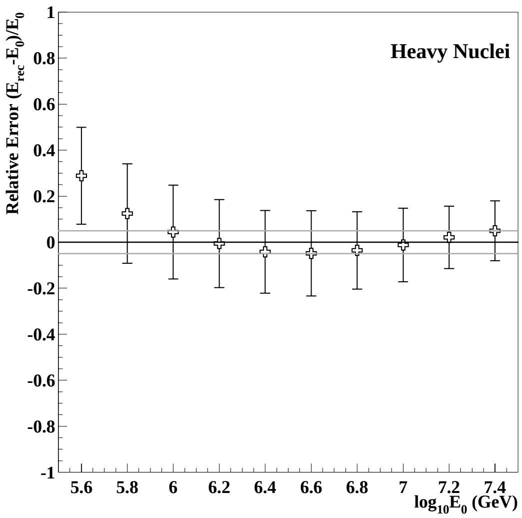

Figure 5: *Primary energy estimation of heavy nuclei (Si,Fe) (MAKET)*

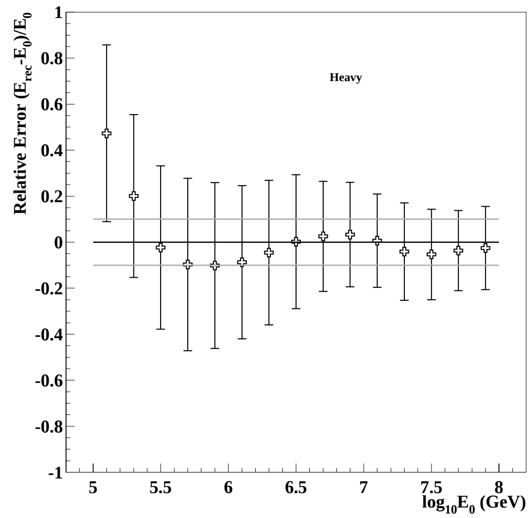

Figure 6: *Primary energy estimation of heavy nuclei (Si,Fe) (KASCADE)*

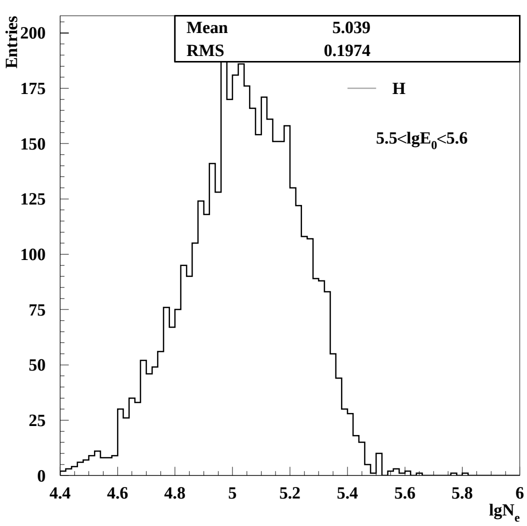

Figure 7: N_e *distribution of MAKET MC protons for the narrow energy interval*

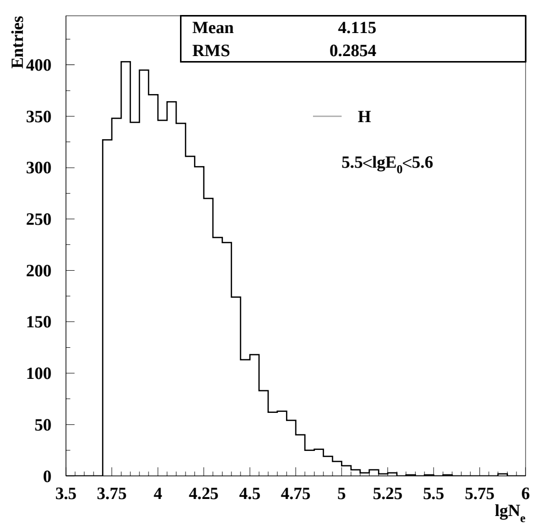

Figure 8: N_e *distribution of KASCADE MC protons for the narrow energy interval*

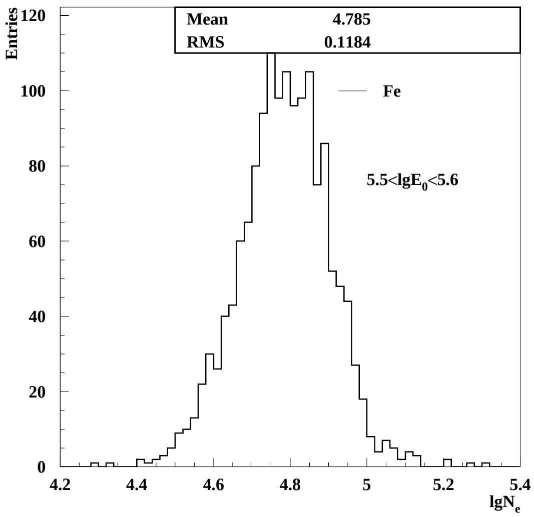

Figure 9: N_e *distribution of MAKET MC irons for the narrow energy interval*

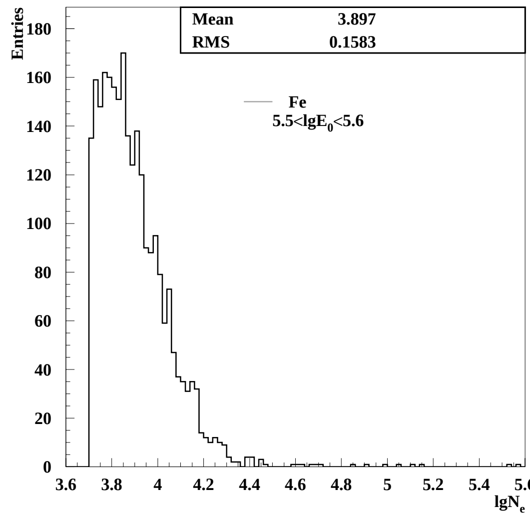

Figure 10: N_e *distribution of KASCADE MC irons for the narrow energy interval*

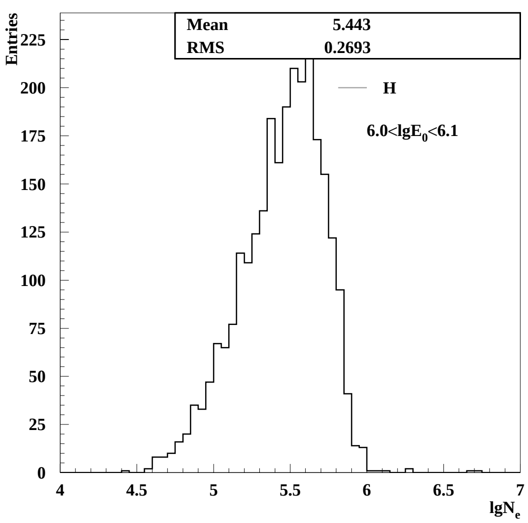

Figure 11: N_e *distribution of MAKET MC protons for the narrow energy interval*

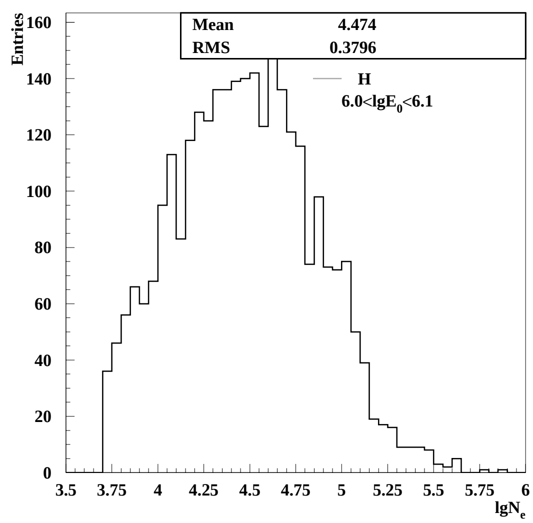

Figure 12: N_e *distribution of KASCADE MC protons for the narrow energy interval*

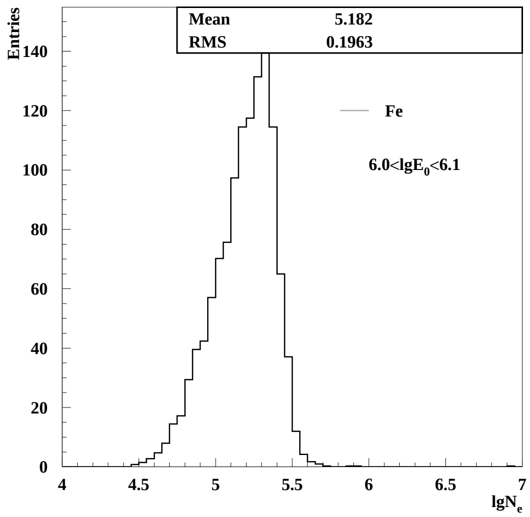

Figure 13: *N_e distribution of MAKET MC irons for the narrow energy interval*

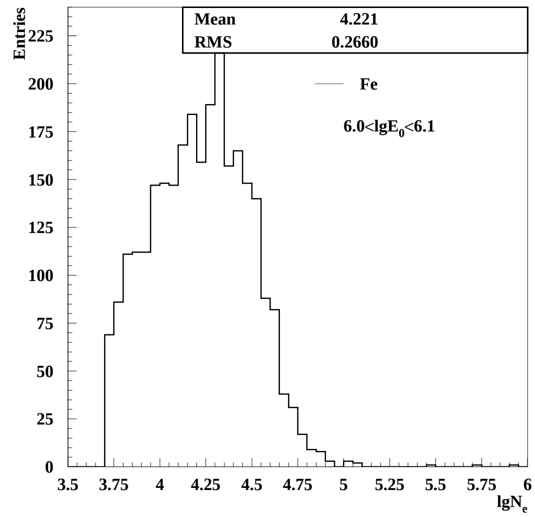

Figure 14: *N_e distribution of KASCADE MC irons for the narrow energy interval*

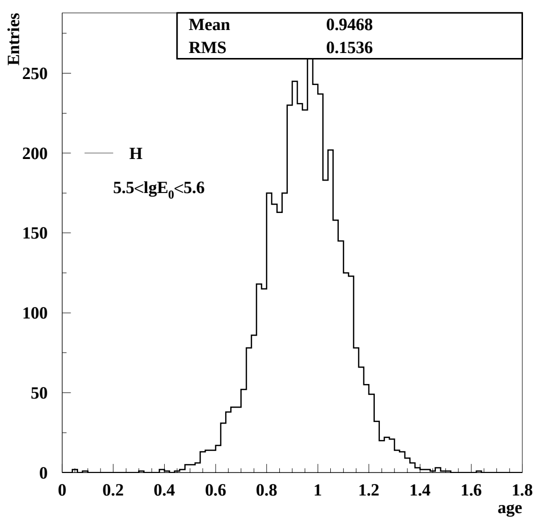

Figure 15: *Age distribution of MAKET MC protons for the narrow energy interval*

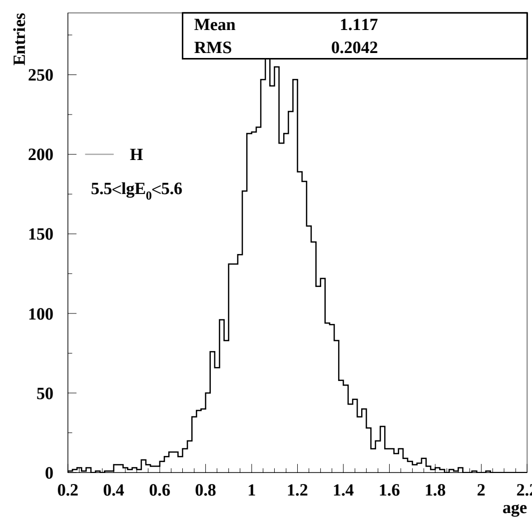

Figure 16: *Age distribution of KASCADE MC protons for the narrow energy interval*

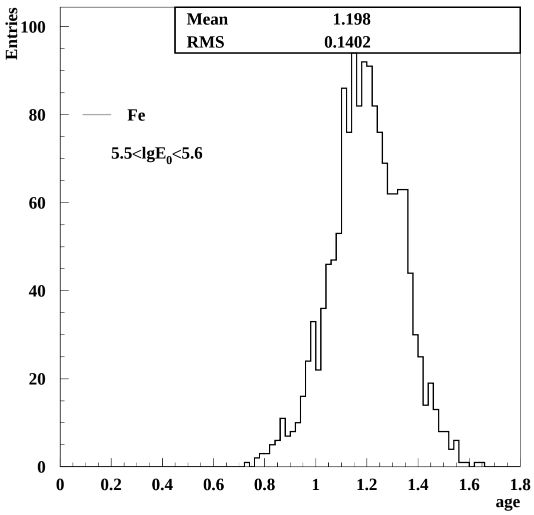

Figure 17: *Age distribution of MAKET MC irons for the narrow energy interval*

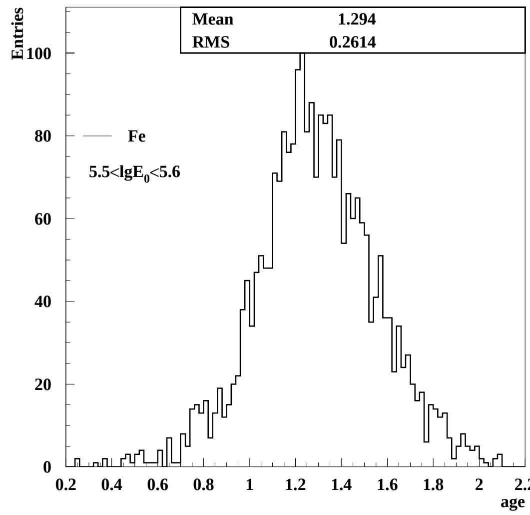

Figure 18: *Age distribution of KASCADE MC irons for the narrow energy interval*

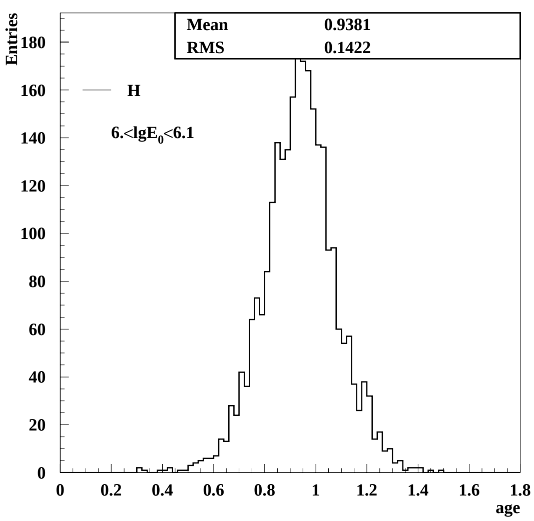

Figure 19: *Age distribution of MAKET MC protons for the narrow energy interval*

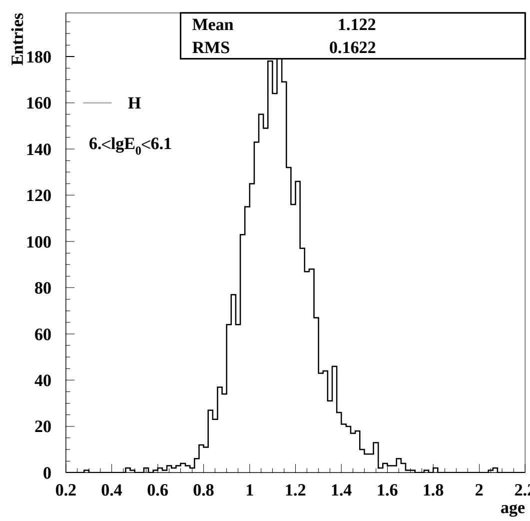

Figure 20: *Age distribution of KASCADE MC protons for the narrow energy interval*

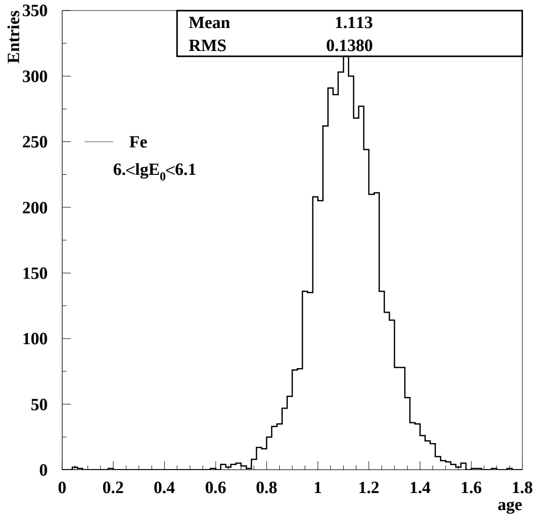

Figure 21: *Age distribution of MAKET MC irons for the narrow energy interval*

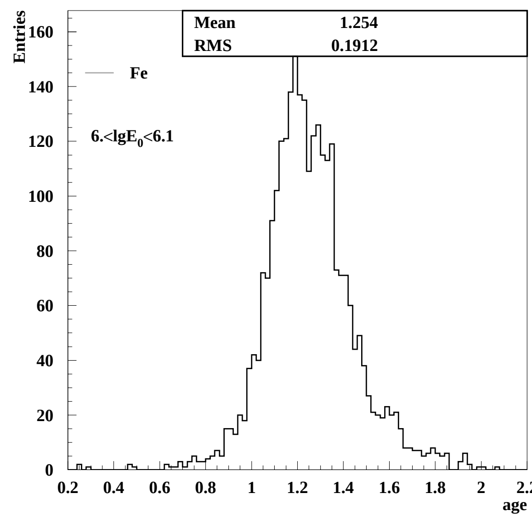

Figure 22: *Age distribution of KASCADE MC irons for the narrow energy interval*

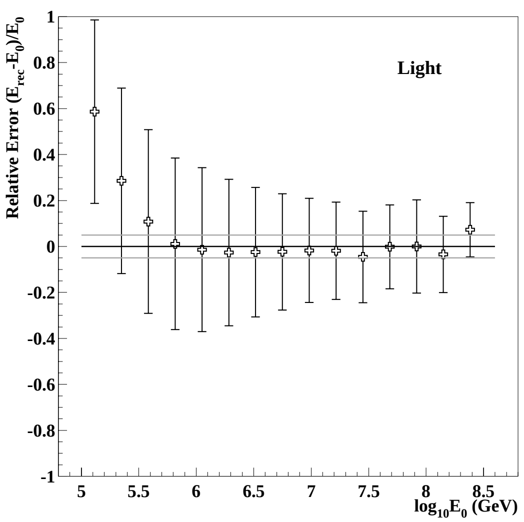

Figure 23: *Primary energy estimation of light nuclei (H,He) by N_e, N_μ^{tr} and age parameters (KASCADE)*

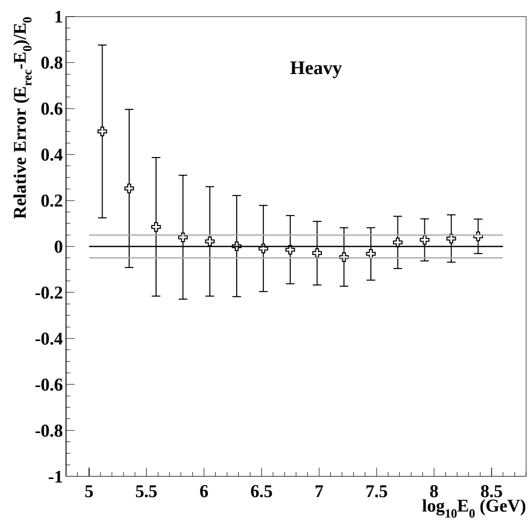

Figure 24: *Primary energy estimation of heavy nuclei (Si,Fe) by N_e, N_μ^{tr} and age parameters (KASCADE)*

Table 7: Neural classification into two classes using N_e, N_μ^{tr} and *age* parameters for KASCADE

	light	heavy
light	0.805	0.195
heavy	0.160	0.840