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A.A.CHILINGARIAN

ON THE EFFICIENCY OF THE HIGH-ENERGY PARTICLE IDENTIFICATION STATISTICAL METHODS

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An attempt is made as to analyze the statistical methods of making decisions on the high-energy particle identification. The Bayesian approach is shown to provide the most complete account of the primary discriminative information between the particles of various types, it does not impose rigid requirements on the density form of the probability function and ensures the account of the a priori information as compared with the Neyman-Pearson approach, the minimax technique and the heristic rules of the decision limits construction in the variation region of the specially chosen parameter The methods based on the concept of the nearest neighbourhood are shown to be the most effective one among the local methods of the probability function density estimation. The probability distances between the training sample classes are suggested to make a decision on selecting the high-energy particle detector optimal parameters. The method proposed and the software constructed are tested on the problem of the cosmic radiation hadron identification by means of transition radiation detectors (the "PION" experiment).

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ОБ ЭФФЕКТИВНОСТИ СТАТИСТИЧЕСКИХ МЕТОДОВ ИДЕНТИФИКАЦИИ ЧАСТИЦ ВЫСОКИХ ЭНЕРГИЙ

В работе предпринята попытка анализа статистических методов принятия решений о идентификации частиц высоких энергий. Показано, что в сравнении с подходом Неймана-Пирсона, мини-максной методикой и эвристическими правилами построения границ решения в области изменения специально выбранного параметра, байесовский подход обеспечивает наиболее полный учет первичной различительной информации между частицами разных типов, не налагает жестких требований на вид плотности функций правдоподобия и обеспечивает учет априорной информации. В работе показано, что среди локальных методов оценки плотности вероятности функции правдоподобия наиболее эффективны методы, основанные на концепции "ближайшего соседства". Для принятия решения о выборе оптимальных параметров детекторов частиц высоких энергий предложено использовать вероятностные расстояния между классами обучающей выборки. Предлагаемые методы и созданное математическое обеспечение проверены на задаче идентификации адронов космического излучения с помощью детекторов переходного излучения (эксперимент "Пион").

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Introduction

The spectrometric and shower installations as well as the Čerenkov counters are used for the high-energy particle identification. However, at very high energies (> 1 TeV) it is difficult to achieve the high accuracy of identification at reasonable losses, therefore the methods based on the registration of the transitions radiation occurring at the relative particle passage through the laminar medium have become widespread [1].

The peculiarity of the identification problems consists in the impossibility to confine oneself to various hypotheses realization probability predictions - the final choice of the decision on the particle type is required [2]. The algorithm the decision choice is realized with is called a decision rule or a classifier.

The main premise of the classification theory methods application to the experimental information processing problem is the existence of the experimental installation imitation model and the randomness of the output variable [3]. The qualitative judgement on the registered particle type is the output variable in the identification problems.

The statistical decision optimization is connected with the choice of the decision rules minimizing the identification errors and losses due to them.

Assume that the space of the possible states of nature (or simply states) consists of two mutually exclusive events A and B. The event A - a particle of type a traversed the installation, the event B - a particle of type b. The space of the possible statistical decisions will also consist of two elements $\tilde{A}$ and $\tilde{B}$, where $\tilde{A}$ is the decision on that the A type event is realized, $\tilde{B}$ - the B type event. The relationship between the state and decision spaces is realized by means of the observations of the experiment outcomes.

The Z outcomes space comprises all the possible values measured in the experiment if the A and B events are realized. The decision rule puts a definite element of the decision space in correspondence with each element of the Z set.

$$d(z) = \begin{cases} A & \text{- if a definite condition holds} \\ B & \text{- in the opposite case} \end{cases}$$

for all the $z \in Z$

If one can enclose the set of values answering the a and b type particles in the nonintersecting regions, then it is possible to formulate a condition providing the correct classification. However the situations are encountered much more frequently when the distributions of the measured values overlap more or less significantly, then the following event combinations are possible:

$$A\tilde{A},\ B\tilde{B},\ A\tilde{B},\ B\tilde{A}$$

The first two combinations correspond to the correct identification,

the last two ones lead to the errors and losses due to them arising when the type b is attributed to the particle a and vice versa.

The principle impossibility as to reduce simultaneously both the errors to the desired level leads to some subjectivity in the optimal statistical decision selection.

According to the most wide-spread technique of Neyman-Pearson [4], the error of the most important event is fixed at the minimum level (usually 1%, 5%). The rules minimizing the alternative error (the one of the second kind) will be the optimal decision rules in this approach. Another approach is connected with the selection of the decision rules minimizing the largest possible error [5]. The Neyman-Pearson approach from the very beginning introduces an asymmetry into the space of the possible decisions, the minimax approach protects from very large losses, but in the meanwhile the possible compromise between the risk and advantage is lost.

The most general approach of the statistical decisions optimization (the Bayesian approach) formalizes the account of all the losses expected. The total account of all the information available concerning the experiment, both the one known before the experiment - the a priori information, and the one obtained after the experiment - the posterior information, is the purpose of the optimization analysis from the Bayesian positions. The Bayesian decision rules are constructed proceeding from a functional minimization representing the losses averaged in all the possible event combinations. There are no decision rules better than the Bayesian one for all the possible events simultaneously [6].

Recently, the nonparametrical decision rules [7, 8] were suggested to interpret the data from the cosmic ray physics experimental installations with the most evident advantage consisting in the total account of the dis-

criminative information between the events of various types. The nonparametrical decision rules are based on the probability density function local estimation, they do not use any assumption as to its form, operate with the primary experimental information and do not deal with the heuristic choice of any parameter. The desire to demonstrate that the Bayesian parametrical procedures are simple and economical in the performance and provide the most reliable identification, is the purpose of the present analysis.

1. The Bayesian Decision Rules

The Bayesian approach implies the determination of the probability and cost measures on the state, decision and experiment outcome spaces. There are several ways of constructing such measures ϑ, the most wide-spread one of which being as follows:

1) The cost measure $v(A, B)$ is determined on the direct product of the state and decision spaces - the losses in the case of the $\tilde{B}$ decision, the state of nature is A. For the identification problems one may assume the losses to be zero at the correct classification and identical at any error - the so-called simple function of losses:

$$v(A, \tilde{A}) = v(B, \tilde{B}) = 0 \qquad (1.1)$$

$$v(A, \tilde{B}) = v(B, \tilde{A}) = 1$$

2) The measure $P(A)$ is determined on the space of the possible states - an a priori probability of the A type event realization, in our case - the portion of the a type particles in the primary flux measured during the previous most reliable experiment. If such experiments were not performed earlier, then the a priori distribution may be assumed uniform:

$$P(A) = P(B) = 1/2 \qquad (1.2)$$

The a priori probabilities may be considered also as the means of "weighing" the errors.

3) The conventional measure (the likelihood function) $p(Z/A)$ is determined on the space of the experiment outcomes - the Z value observation probability, if a is the type of the particle *. The peculiarity of the procedure of the statistical decisions on identification is that the conventional measure is set by means of the imitation experiment results. The way of the likelihood function estimation by the classified outcome sets (the training samples) is the key one in the problem of the statistical decisions and will be treated in the next paragraph.

The information obtained in course of the experiment is summarized in the likelihood function, the information known before the experiment - in the a priori density. One may accumulate all the information available (by means of the Bayesian theorem) and introduce the so-called posterior measure 10

$$P(A/Z) = Cp(Z/A)p(A) \tag{1.3}$$

The posterior density shows the probability of that the A event took place, provided the value Z was observed in the experiment. The normalizing multiplier C is introduced for the posterior measure to be probabilistic (i.e. to have the meaning of the probability density).

Using the condition **

$$P(A/Z) + P(B/Z) = 1 \tag{1.4}$$

---

* In the general case Z is the vector value $\vec{Z} = (Z_1, Z_2, \ldots Z_n)$

** The A and B events form a complete system.

we obtain

$$1/c = p(A)p(z/A)+p(B)p(z/B) \tag{1.5}$$

At the Bayesian approach the average losses are calculated for all the points of the decision space. The average losses (or the risk R) of making decisions at the points $\tilde{A}$ and $\tilde{B}$ have the following form:

$$R(\tilde{A}) = v(A,\tilde{A})p(A/z)+v(B,\tilde{A})p(B/z) \tag{1.6}$$

$$R(B) = v(B,\tilde{B})p(B/z)+v(A,\tilde{B})p(A/z)$$

When using the simple function of losses (1.1)

$$R(\tilde{A}) = p(B/z) \tag{1.7}$$

$$R(\tilde{B}) = p(A/z)$$

The risk to make a decision A according to the outcome obtained is equal to the B event posterior probability. The risk to make a decision B - to the A event posterior probability. Hence, the decision rule minimizing the risk at all the points z of the experiment outcome space will look like

$$d(z) = \begin{cases} \tilde{A} & \text{if } p(A/z) > p(B/z) \\ \tilde{B} & \text{in the opposite case} \end{cases} \tag{1.8}$$

The rule (1.8) is called the rule of the posterior density maximum. In observing the z outcome the particle is related to the type ensuring the posterior density maximum at the point z. Making use of the ratio (1.3) one may obtain the Bayesian decision rule of the likelihood function maximum:

$$d(z) = \begin{cases} \tilde{A} & \text{if } p(A)p(z/A) > p(B)p(z/B) \\ \tilde{B} & \text{in the opposite case} \end{cases} \quad (1.9)$$

If the a priori probabilities are equal, then

$$d(z) = \begin{cases} \tilde{A} & \text{if } p(z/A) > p(z/B) \\ \tilde{B} & \text{in the opposite case} \end{cases} \quad (1.10)$$

The event is referred to the type ensuring the likelihood function maximum at the point Z .

2. The Likelihood Density Function Local Estimations

As we have seen, the Bayesian decision rules are based on the calculation of the likelihood function density. Assuming the latter to be known, one may directly make a statistical decision after calculating its value for various particle types and compare the obtained magnitudes.

However, as it was already mentioned, all the information on the likelihood function is contained in the training samples - the imitation experiment realization sets for the particles of various types.

If the likelihood function form is known, the experimental information vector dimensionality is not large and (or) the condition of the conventional density parametrical independence holds:

$$p(\vec{z}/A) = p_1(z_1/A) \times p_2(z_2/A) \times \cdots p_n(z_n/A) \quad (2.1)$$

then the maximum likelihood method may be applied to estimate the likelihood function unknown parameters. If the condition (2.1) does not hold, then the computational difficulties due to the account of the correlations between the $\vec{z}$ vector components make the application of the maximum likelihood

method inefficient.

The most general (nonparametrical) approach not connected with any simplifying assumptions and requiring only the continuity of the likelihood function, is based on the density local estimation at the points $\vec{Z}$ of the experiment outcomes space 11 .

The density local estimation is based on the following assumptions:

1. The space of the experiment outcomes may be divided into Ωi regions containing all the possible realization of the experiment.

2. The probability of hitting the region Ωi is equal to the ratio K_i/N , where K_i is the number of outcomes involved in the i -th region, N is the total number of outcomes.

3. the arbitrary distance measure may be introduced into the outcome space.

In case of items 1-3 being true we shall obtain the local density estimation in the Ωi region in the following form:

$$p_i(\vec{Z}/A)_N = K_i/N\phi_i \qquad (2.2)$$

where ϕ_i is the Ωi region volume.

In dividing the space into rectangular cells of identical sizes (the hystogram method with the constant spacing) the local estimation is connected with considerable difficulties.

Indeed, let us assume the vector $\vec{Z}$ dimensions to be equal to 5. To ground statistically the hystogram method at least 5 values are required to get into each cell 12 . If dividing the variation region of the $\vec{Z}$ vector each component into at least ten intervals, then the total number of the cells will amount to 10^5 and the imitation experiments minimal number - to 5×10^5.

The statistical provision of various cells is not identical and the density inside the N-dimensional cube (cell) may vary considerably.

Much more attractive is the idea to construct the density estimations using the N-dimensional hyperball with the centre at the point of interest. In this case there is no need to keep in memory the "density library", each time the densities will be calculated anew. The asymptotic properties of the estimations are such, that one may limit himself to a much less number of training sample sizes than in the first case [3]. The Ω_i region may vary, for the density estimations at each point to be identically provided statistically - e.g. the regions may be required to contain equally K vectors from the training sample. The Ω_i region volume will thus depend on the space dimensions, the training sample size, the K parameter and the location of the $\vec{z}$ hyperball centre.

There is a large variety of the density nonparametrical estimations, the principal ones differing in the way of the training sample application. The first rule of the KNN * [4] does not make any difference between the training sample classes. The Ω_i regions are constructed so as to involve K members of the united sample, the density estimation is obtained in the following form:

$$P_i(\vec{z}/A) = K_a/N_a \times \phi_i \qquad (2.3)$$

$$P_i(\vec{z}/B) = K_b/N_b \times \phi_i \qquad K_a + K_b = K$$

ϕ_i is the Ω_i region volume

N_a and N_b are the total numbers of the training sample alternative classes. K_a and K_b are the numbers of representatives of the A and B classes among the K nearest neighbours of the $\vec{z}$ vector.

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* KNN - K nearest neighbours.

The second rule KNN 14 implies the construction of various regions Ω_{iA} and Ω_{iB}, each involving K members of various training sample classes:

$$P_i(\vec{Z}/A) = K/N_A \times \Phi_{iA} \qquad (2.4)$$

$$P_i(\vec{Z}/B) = K/N_B \times \Phi_{iB}$$

Φ_{iA} and Φ_{iB} are the Ω_{iA} and Ω_{iB} regions volumes.

The second rule is usually used in the modified variant by calculating and then averaging the density for various K 15 . The K parameter value is determined by the training sample sizes and the true densities. Usually the optimal K is equal to the square root of the training sample size:

$$K_{opt} \approx \sqrt{N} \qquad (2.5)$$

In the computational aspect the distance ranging from the studied point to all the training sample members is a process that requires the most time expenditure in the KNN rules.

There is no need to calculate the probability densities in the explicit form when classifying, as in the Bayesian decision rules it is enough to know only which of the alternative classes ensures the highest density.

Substituting the density values (2.4) and (2.5) in the Bayesian decision rules maximizing the posterior risk and the likelihood function (1.8), (1.9), we shall obtain the KNN_1 and KNN_2 decision rules*:

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* The relation between the KNN decision rules and the Bayesian ones is analogous to that between the optimal estimations and the general population parameters, therefore the KNN rules are sometimes called "empirical Bayesian rules".

$$d_K(\vec{z}) = \begin{cases} \tilde{A} & \text{if } K_A > K_B \\ \tilde{B} & \text{in the opposite case} \end{cases}, \quad \begin{matrix} P(A) = P(B) \\ K_A + K_B = K \end{matrix} \qquad (2.6)$$

$$d_K(z) = \begin{cases} \tilde{A} & \text{if } R_{KA} < R_{KB} \\ \tilde{B} & \text{in the opposite case} \end{cases}, \quad P(A) = P(B) \qquad (2.7)$$

R_{KA} and R_{KB} are the distances to the K-nearest neighbour for the classes A and B.

The second decision rule of the nearest neighbourhood KNN_2 allows one to take account of the a priori probabilities directly:

$$d_K(\vec{z}) = \begin{cases} \tilde{A} & \text{if } P(A)/R_{KA} > P(B)/R_{KB} \\ \tilde{B} & \text{in the opposite case} \end{cases} \qquad (2.8)$$

Both the Euclidean and the special metrics taking account of the data structure may be used as a distance function 16 .

3. Hadron Identification by Energy Release in the Transition Radiation Detector

The "PION" installation has been functioning in the high-altitude station "Aragats" of the Yerevan Physics Institute ever since 1977, one of its purposes being the cross section measurement of the cosmic radiation flux hadron interaction with various nuclei. Five trays of the transition radiation counters are used to identify protons and π -mesons, each consisting of a radiator and a multiwire proportional chamber 17 .

To calculate the expected energy release of pions and protons a detailed imitation model of the installation is constructed 18 .

The imitation programme was used to form the training sample that would help to outline the decision boundaries in the variation space of the choser

parameter. These boundaries determined the regions (the point sets) of decision making and the region of uncertainty. One may achieve the great reliability of the identification by enlarging the uncertainty region, but this leads to reducing the detection efficiency because of the refusal to make a statistical decision when the parameter hits the uncertainty region. The solution of this contradiction between the reliability and efficiency, typical of the statistical decisions problem depends in many respects on the identification parameter chosen.

The geometric mean is one of the first parameters suggested. This parameter used allows one to cut off somewhat the distribution tails as compared with the arithmetic mean and hence to contract the uncertainty region boundaries [19], but the scalar $Z = \sqrt{Z_1 \times Z_2 \times \ldots Z_n}$ substitution for the vector response $\vec{Z} = (Z_1, Z_2, \ldots Z_n)$ results in the considerable loss of the discriminative information.

Ref. [20] shows that the use of the information from all the installation trays allows one to enhance significantly the reliability of the statistical conclusions. The parameter that takes into account the information from all the trays of the transition radiation detector is the likelihood ratio

$$L(\vec{z}) = \hat{P}_{\pi}(\vec{z}/\pi)\ P_p(\vec{z}/p) \tag{3.1}$$

where $\hat{P}_{\pi}$ and $\hat{P}_p$ are the local estimations of the joint distribution density of the values $Z_1, Z_2 \ldots, Z_n$, provided that the particle releasing the energy is a pion or a proton, respectively.

The likelihood functions P_{π} and P_p are estimated by the training samples obtained in the imitation or calibration experiments. The estimations are carried out by the hystogram methods in the conventional parametrical independence approximation of the likelihood function:

$$P_{\pi}(\vec{Z}/\pi) = P_{\pi_1}(Z_1/\pi) \cdot P_{\pi_2}(Z_2/\pi) \cdot \ldots P_{\pi_n}(Z_n/\pi)$$
$$P_{P}(\vec{Z}/P) = P_{P_1}(Z_1/P) \cdot P_{P_2}(Z_2/P) \cdot \ldots P_{P_n}(Z_n/P) \qquad (3.2)$$

The application of the Bayesian approach, in particular the nonparametrical decision rules based on the concept of the nearest neighbourhood, to the problem of the pion and proton identification, does not impose rigid requirements (of the (3.2)type) on the distribution function density type and estimates the local density more efficiently.

The technique of calculating was as follows:

1. The formation of the training sample, consisting of the imitation programme 18 realizations.
2. The identification of the "pseudoexperimental" vector sets - also the simulation programme realizations.
3. The determination of the percentage error in classifying the "proton" and "meson" events.

The calculations were executed with the use of the first and second decision rules KNN (2.6), (2.8). The training sample size, the classifiers parameters, the number of the installation trays, the energy of particles all these varied.

Besides, the distance probability measures between the training sample classes were calculated.

Via Bhattacharya distance 21

$$\rho_B = -\ln \sum_{-\infty}^{\infty} \sqrt{P_{\pi}(\vec{Z}/\pi)\, P_{P}(\vec{Z}/P)}\, dZ \qquad (3.3)$$

one can express the upper and lower limits of the classification expected error

$$\varepsilon_B \leq 1/2 \exp(-\rho_B)$$

$$\varepsilon_H \geq 1/2 - 1/2 (1 - 4\varepsilon_B)^{1/2} \qquad (3.4)$$

In the general case the integral (3.3) is calculated by the numerical methods. If the distributions functions are close to the normal ones and the covariance matrices of both the training sample classes coincide, then the Bhattacharya distance coincides with the Mahalonobis one 22 , that can be calculated analytically

$$\rho^2 = (\vec{\mu}_p - \vec{\mu}_\pi)^T \Sigma^{-1} (\vec{\mu}_p - \vec{\mu}_\pi) \qquad (3.5)$$

where $\vec{\mu}_p$ and $\vec{\mu}_\pi$ are the averages of the training sample classes, Σ is the common covariance matrix.

Figure 1c, d presents the probability distance dependences on the particle energy and the number of the transition radiation detector trays. Naturally, the separability of the training sample classes improves and the probability distance enlarges with the increase of the experimental information vector dimensionality. The construction of the detector was chosen so, that the largest distance and hence the best discrimination are achieved at the energy 1000 GeV.

Figure 1a, b shows the corridors calculated by formulae (3.5), where the classification expected error * must be involved.

In registrating the transition radiation by 5 trays of proportional chambers the classification expected error is equal to $\approx$ 10% (the energy 1000 GeV), that allows one to classify reliably the cosmic radiation protons and pions. The results of the classification by the KNN_1 and KNN_2 decision

* The average error $(P(\pi\tilde{p}) + P(p\tilde{\pi})/2$ is meant, the cost of misclassification coincides with its probability when using the simple loss function.

rules (the training sample size is equal to 500, the control sample size - to 500, too) are presented in the same figures. It is obvious that the KNN_2 rule provides noticeably better results than the KNN_1 one, due to a more precise estimation of the likelihood function.

The dependences of the efficiency and error of protons and pions identification on the K parameter in the KNN_1 and KNN_2 rules are presented in Fig.2. The training samples sizes are equal to 200. The classification results show that the training samples volume is sufficient and that the algorithms convergence is attained at $K \approx \sqrt{N}$

Fig.3 shows the results of the KNN_2 rule comparison at K = 13 with the curves of the efficiency dependence on the classification expected error, the geometric mean 17 applied as a parameter. At the error fixed, one may achieve a higher efficiency of registration in case the nonparametrical decision rule is applied. If, e.g. one limits the error $P(\pi\,\tilde{P})$ to 10%, then the pion detection efficiency will be equal to 75 and 85%, respectively (in detecting by four trays).

Conclusion

The statistical decision optimization in the high-energy particle identification problems is achieved in the following ways:

1. By applying the Bayesian decision rules, that minimize the expected average losses and take account of all the discriminative information between the particles of various types and of all the a priori information available.
2. By the likelihood function nonparametrical local estimation, allowing one to reduce substantially the training samples sizes and to enhance the estimation accuracy due to the special selection of the hystogram cell

forms.

3. By applying the distance probability measures between the training sample classes for the purposes of planning the experiment.

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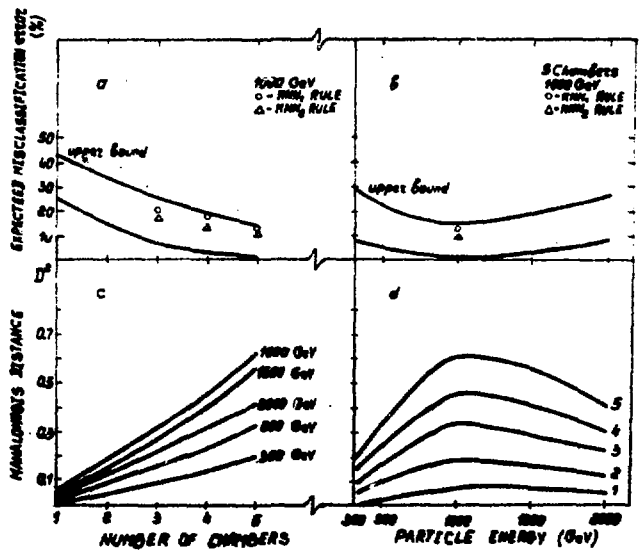

Fig.1. The probability distances between the training sample classes, the limits of the expected misclassification.

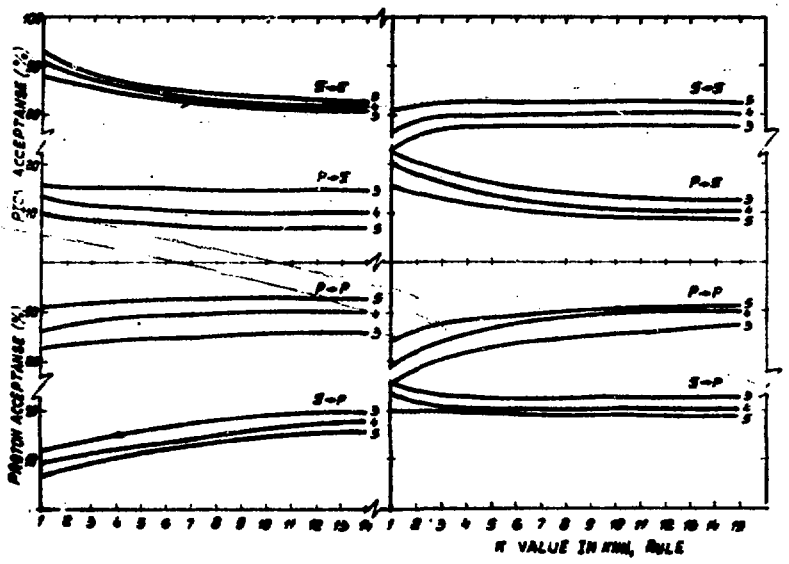

Fig.2. The efficiencies and errors of π -meson and proton identification.

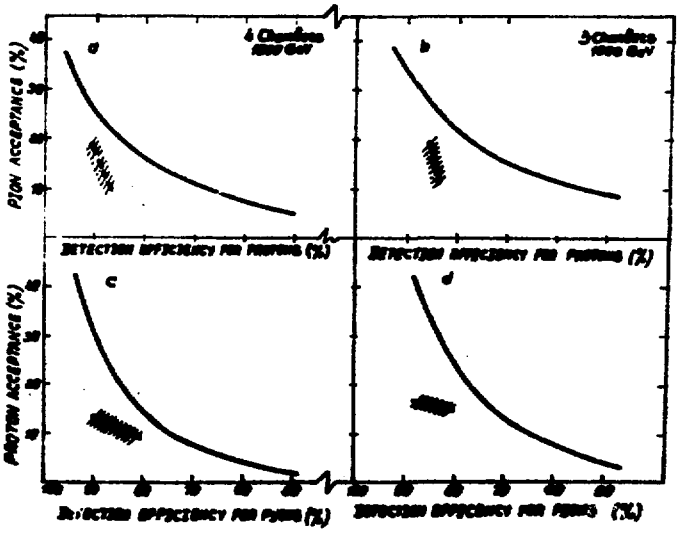

Fig.3. The comparison of the identification techniques.

REFERENCES

1. Алиханян А.И., Обзор работ по рентгеновскому переходному излучению и детекторам на его основе, выполненные в СССР. - Труды Международного симпозиума по переходному излучению частиц высоких энергий, Ереван, 1977, с.41-79.
2. **Cox D.R., Hinkley D.V. Theoretical Statistics. Chapman and Hall, London, 1974, p.453-503.**
3. Круг Г.К., Кабанов В.А., Фомин Г.А.и др. Планирование экспериментов в задачах нелинейного оценивания и распознавания образов. М.: Наука, 1981, с.107-109.
4. Брауили К.А. Статистическая теория и методология в науке и технике. М.: Наука, 1977, с.77-122.
5. Леман Э., Проверка статистических гипотез М.: Наука, 1979, с.21-23.
6. **De Groot M.H. Optimal Statistical Decisions. McGraw-Hill Company, New York, St. Louis, San Francisco, Sydney, Toronto, Mexico, Panama, 1970, p.125-159.**
7. **Chilingarian A.A., Ter-Antonian S.V. On the Possibility of Improving the Multi-Layered Detector Energy Resolution.- Scient.Report EFI-519(6)-82, 1982.**
8. **Chilingarian A.A. On the New Methods of Roentgen-Emulsion Chamber Data Treating.- Scient.Report EFI-544(31)-82, 1982.**
9. **Raiffa H., Schlaifer R. Applied Statistical Decision Theory. M.I.T. Press, Cambridge, Massachusetts and London, p.12-37.**
10. **Bard Y. Nonlinear Parameter Estimation. Academic Press, New York, San Francisco, London, 1979, p.38-43.**
11. **Patrich E.A. Fundaments of Pattern Recognition. Prentice-Hall, INC,**

Englewood Cliffs, N.J., 1972, p.135-240, 180.

12. Статистические методы в экспериментальной физике./ Под редакцией Тяпкина А.А. М.: Атомиздат, 1976, с.251-255.

13. Levine A., Lustich L., Saltzberg B. The Nearest Neighbour Rule for Small Samples Drawn from Uniform Distributions.- IEEE Trans. Inform. Theory, 1973, vol. IT-19, p.697-699.

14. Фукунага К. Введение в статистическую теорию распознавания образов М.: Наука, 1979, с.167-184.

15. Yunch T.R., Tuteor F.B. Comparison of Decision Rules for Automatic EEG Classification.- IEEE Trans. Pattern Analysis Machine Intell., vol. PAMI-2, p.420-428.

16. Dubussion D., Lavison R. Surveillance of a Nuclear Reactor by Use of a Pattern Recognition Methodology.- IEEE Trans. Syst. Man and Cybernet., 1980, vol.SHC 10, p.603-609.

17. Авакян Э.В., Авундян А.Т., Антонян К.Г. и др. Установка для исследования взаимодействий космических адронов с энергией выше 500 ГэВ. Труды Международного симпозиума по переходному излучению частиц высоких энергий. Ереван, 1977, с.277-286.

18. Авундян А.Т., Казарян С.Л., Оганесян А.Г. Алгоритм моделирования РПД детекторов и некоторые его применения. Препринт, ЕФИ-437(44)-80, Ереван 1980.

19. Yuan L.C., Uto H., Dell G.F. and Alley R.W. Relativistic Particle Separation by Means of Transition Radiation Utilizing Multiwire Proportional Chambers.- Phys.Lett., 1972, vol.40B, p.689-698.

20. Cherry M.L., Muller D., Prince T.A. The Efficient Identification of Relativistic Particles by Transition Radiation.- NIM, 1974, vol.115, p.141-150.

21. Heydorn R.H. An Upper Bound Estimate on Classification Error.- IEEE Trans. Inform. Theory, 1968, vol.IT-14, p.783-784.

22. Yablow M., Chu J.T. The Relationship of the Bayes Risk to Certain Separability Measures in Normal Classification.- IEEE Trans. on Pattern Analysis and Machine Intell., 1980, vol.PAMI-2, p.97-100.

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